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WHITE'S GRAMMAR SCHOOL ALGEBRA

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GRAMMAR SCHOOL ALGEBRA

AN INTRODUCTION TO ALGEBRA FOR BEGINNERS

BY

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PREFACE

THE main design of these first lessons in algebra is to afford pupils training in the algebraic solution of problems which may also be solved by analysis or other arithmetical processes, thus correlating arithmetic and algebra in a practical manner. Exercises are also given in the fundamental processes with algebraic numbers both integral and fractional in form, the algebraic processes being similar to the corresponding processes in arithmetic.

Only so much of algebraic notation is given as is necessary in the use of the equation in solving problems and in the more elementary processes. Care has been taken not to include technical matter that properly belongs to text-books for use in high schools. Technical algebra has no value that justifies its introduction before arithmetic is properly completed. The practical value of algebra when taught in connection with arithmetic is almost wholly limited to the use of the algebraic method in the solution of problems and the statement of formulas, and some resulting

familiarity with the expression of numbers by letters and operations by signs.

These introductory lessons in algebra may be readily mastered in the last year of the grammar school. They may be given in one or two exercises each week during the year, or, what is better, they may take the place of arithmetic the last half of the year. They will be found not only an excellent preparation for the study of elementary algebra in the high school, but also of special interest and value to pupils whose schooling may end with the elementary school.

The problems may also be solved by analysis, thus obviating the use of a separate mental arithmetic in the eighth school year.

COLUMBUS, OHIO.

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GRAMMAR SCHOOL ALGEBRA

ALGEBRAIC EQUATIONS

1. If a denotes a certain number, $3a$ denotes 3 times the number; $4a$ denotes 4 times the number; and so on.

1. If a denotes a number, what does $2a$ denote? $5a$? $7a$? $9a$? $12a$?

2. If a denotes a number, what will denote 3 times the number? Five times the number? Seven times the number? Ten times the number?

3. If x denotes the number of feet in a yard, what will denote the number of feet in 2 yards? 4 yards? 6 yards? 8 yards? 10 yards?

4. If x denotes the number of hours in a day, what will denote the number of hours in 3 days? 5 days? 7 days? $\frac{2}{3}$ of a day? $\frac{3}{4}$ of a day?

5. If x denotes a man's age, what will denote 3 times his age? 5 times his age? $\frac{2}{3}$ of his age? $\frac{3}{4}$ of his age?

6. If x denotes the number of pounds in a bushel of wheat, what will denote the number of pounds in 7 bushels? In $2\frac{1}{2}$ bushels?

It is thus seen that numbers may be expressed by letters as well as by figures.

2. In algebra the signs $+$, $-$, $\times$, $\div$, and $=$ have the same meaning as in arithmetic. The expression $7 + 5$ denotes that 5 is to be added to 7, and $7 - 5$ denotes that 5 is to be subtracted from 7.

In like manner, $a + b$ denotes that the number represented by b is to be added to the number represented by a, and $a - b$ denotes that the number represented by b is to be subtracted from the number represented by a.

1. The sum of $3a$ and $4a$ is expressed by $3a + 4a$, which is $7a$. How many times a in the sum? How many times a in the sum of $5a$ and $3a$?

The number that denotes how many times a letter is taken is called its **coefficient**. Thus, in $7a$, the 7 is the coefficient of a.

2. Express by the sign $+$ the sum of $4x$ and $6x$. How many times x in the sum?

3. Express by the sign $+$ the sum of $2x$, $3x$, and $5x$. How many times x in the sum?

4. If x denotes A's age and $3x$ B's age, what will express the sum of their ages? How many times x in the sum?

5. If x denotes the cost of a chair, $3x$ the cost of a table, and $5x$ the cost of a lounge, what will express the cost of the three articles? How many times x in the cost?

6. The difference expressed by $7a - 4a$ is $3a$. How many times a in the difference?

7. What is the difference expressed by $8a-5a$? $9a-4a$? $8a-3a$? $12a-7a$? $13a-8a$? $21a-9a$?

8. If x denotes the cost of a chain and $3x$ the cost of a watch, what will express the difference of their cost? How many times x in the difference?

9. If $5x$ denotes A's age and $3x$ denotes B's age, what will express the difference of their ages? How many times x years is A older than B?

3. The equality of two numbers may be expressed by the sign $=$, which is read "equals" or "is equal to." Thus, $2x+4x=24$ is read "two x plus four x equals 24."

If $4x=20$, $1x$, or x, is $\frac{1}{4}$ of 20, which is 5. The number 5 which x denotes in the equation $4x=20$ is called the **value of** x.

1. If $3x=15$, what is the value of x? Of $4x$? $5x$?
2. If $3x+4x=35$, what is the value of x? Of $6x$?
3. If $3x+4x+5x=60$, what is the value of x?
4. If $7x+5x+3x=45$, what is the value of x?
5. If $12x-5x=28$, what is the value of x?
6. If $7x+4x-3x=32$, what is the value of x?
7. If $8x+7x-6x=54$, what is the value of x?

4. The expression $7+5=3\times4$ denotes that the sum of 7 and 5 is equal to the product of 3 and 4. In like manner, $a+b=c\times d$ denotes that the sum of the numbers represented by a and b is equal to the product of the numbers represented by c and d.

An expression denoting the equality of two numbers is called an **equation.** Thus, $2x + 3x = 25$ is an equation. The number at the left of the sign of equality is called the **first member** of the equation, and the number at the right of the sign is called the **second member.**

An equation in which all the numbers are expressed by figures is called an **arithmetical equation.**

An equation in which one or more of the numbers are expressed by letters is called an **algebraic equation.**

Thus, $7 + 5 = 3 \times 4$ is an arithmetical equation, and $3x + 2x = 20$ and $a - b = c$ are algebraic equations.

5. If $x = 4$, then $x \times 3 = 4 \times 3$, or $3x = 12$; and if $3x = 12$, then $3x \div 3 = 12 \div 3$, or $x = 4$. Hence, if both members of an equation be multiplied or divided by the same number, *their equality will not be affected.*

6. The finding of the value of the unknown number (x) in an equation is called the **solution of the equation.**

Solution of the equation $3x + 5x = 32.$

Adding the terms $(3x + 5x)$, $8x = 32;$

whence (dividing both members by 8), $x = 4.$

The solution of an equation may be *verified* (shown to be correct) by substituting for the unknown number in the equation its value found. If the two members of the equation are equal, the solution is correct.

Thus, substituting 4 for x in $3x + 5x = 32$, we have $3 \times 4 + 5 \times 4 = 32$, or $32 = 32$; hence 4 is the correct value of x.

Solve and verify the following equations:

1. $3x + 4x = 56$.
2. $9x - 6x = 30$.
3. $12x + 3x = 90$.
4. $17x - 9x = 80$.
5. $3x + 4x + 5x = 84$.
6. $6x + 8x - 9x = 45$.
7. $12x - 3x - 4x = 35$.
8. $7x - 6x + 5x = 54$.
9. $5x + 6x - 7x = 40$.
10. $8x - 3x + 4x = 36$.
11. $15x - 6x - 4x = 45$.
12. $20x - 14x + 2x = 64$.
13. $11x - 6x - 3x = 40$.
14. $8x + 5x - 7x = 60$.
15. $10x - 7x + 4x = 28$.
16. $15x - 8x - 2x = 40$.
17. $9x + 3x + 8x = 60$.
18. $17x - 8x - 3x = 36$.
19. $14x + 6x - 8x = 72$.
20. $23x - 9x - 4x = 100$.
21. $8x + 12x - 5x = 45$.
22. $16x - 7x - 3x = 42$.
23. $7x + 13x - 8x = 48$.
24. $18x - 7x - 6x = 60$.

ALGEBRAIC SOLUTION OF PROBLEMS

7. The solution of problems by means of an algebraic equation is called the **algebraic solution** of problems.

The first step in the algebraic solution of a problem is to state the conditions of the problem in the form of an equation, called the **statement**; and the second step is to find the value of the unknown number, called the **solution** of the equation.

The advantage of the algebraic method of solving problems is readily shown by its use in the solution of problems which can also be solved by the methods of arithmetic. Take, for illustration, this problem:

A's age is twice B's age, and the sum of their ages is 60 years. What is the age of each?

Arithmetical Solution

Since A's age is twice B's age, B's age plus twice B's age, or three times B's age, is 60 years; hence B's age is one third of 60 years, which is 20 years, and A's age is twice 20 years, which is 40 years. Hence A's age is 40 years, and B's age 20 years.

Algebraic Solution

Let x = B's age; then $2x$ = A's age.

Hence $x + 2x = 60.$

Adding terms $(x + 2x)$, $3x = 60;$

whence $x = \frac{1}{3}$ of $60 = 20$, B's age,

and $2x = 40$, A's age.

Hence A's age is 40 years, and B's age 20 years.

In the algebraic statement of a problem, each number is considered *abstract*. Thus, since 20 years = 1 year × 20, x in the above solution represents 20, the concrete unit (1 year) being omitted. To express the number of years, the abstract value of x, when found, is considered as multiplied by the omitted concrete unit (1 year); 1 year × 20 = 20 years.

PROBLEMS

Solve the first nine of the following problems by the arithmetical method and also by the algebraic method:

1. A and B together have \$60, and A has 3 times as much money as B. How much money has each?

ARITHMETICAL SOLUTION

Three times B's money = A's money;
then 3 times B's money + B's money = \$60.
Hence 4 times B's money = \$60;
whence B's money = $\frac{1}{4}$ of \$60 = \$15,
and A's money = 3 × \$15 = \$45.
Hence A has \$45, and B \$15.

ALGEBRAIC SOLUTION

Let x = B's money;
then $3x$ = A's money.
Hence $x + 3x = 60$.
Adding terms, $4x = 60$;
whence $x = 15$, B's money,
and $3x = 3 \times 15 = 45$, A's money.
Hence A has \$45, and B \$15.

2. A father and his son together earn \$54 a month, and the father earns twice as much as the son. How much does each earn?

3. A coat and vest cost $24, and the coat cost 5 times as much as the vest. What was the cost of each?

4. The sum of two numbers is 45, and the greater number is 4 times the less. Find the numbers.

5. Divide 36 into two parts such that the greater shall be 3 times the less.

6. Cut a piece of tape 54 yards long into two pieces such that the longer piece shall contain 5 times as many yards as the shorter.

7. A and B together own 150 sheep, and A owns twice as many sheep as B. How many sheep does each own?

8. A father is 3 times as old as his elder son, the elder son is twice as old as the younger, and the sum of all their ages is 63 years. How old is each?

9. If a number be increased by twice itself, the result will be 63. What is the number?

10. Divide $840 among A, B, and C, giving to B twice as much as to A, and to C twice as much as to B.

11. Divide $18 among A, B, and C, giving to A twice as much as to B, and to C twice as much as to A and B together.

12. Three men, A, B, and C, bought a mill for $16000. A paid twice as much as B, and C paid 5 times as much as B. How much did each pay?

13. A school enrolls 240 pupils, and twice the number of boys equals the number of girls. How many of each sex are enrolled?

14. A man sold a horse and a buggy for $180, and received twice as much for the horse as for the buggy. What was the price of each?

15. A farmer who owned a flock of sheep bought 3 times as many sheep as he owned, and then had 248 sheep. How many sheep did he buy?

16. A's age is twice B's, and B's age is twice C's, and the sum of all their ages is 126 years. What is the age of each?

17. Divide a piece of cloth containing 42 yards into three pieces, making the second piece 3 times the length of the first, and the third piece one half of the length of the other two pieces together.

18. An estate of $18000 was bequeathed to a widow and two sons. The sons received equal shares, and the widow twice as much as the two sons together. How much did each receive?

19. Three men, A, B, and C, formed a partnership in business, with a capital of $12000. A furnished twice as much as B, and C as much as A and B together. How much capital did each furnish?

20. In three years a merchant made a profit of $6300. The profit the second year was twice the profit of the first year, and the profit the third year was twice that of the second year. What was the profit each year?

21. A father and his two sons earn $140 a month; the father earns twice as much as the elder son, and the elder son twice as much as the younger. How much does each earn?

22. A farmer raised 720 bushels of grain, consisting of wheat, corn, and oats; he raised twice as much wheat as corn, and 3 times as much oats as corn. How many bushels of each grain did he raise?

23. A banker paid \$102 in ten-dollar, five-dollar, and two-dollar bills, using the same number of bills of each kind. How many bills of each kind did he use?

Let x = number of bills of each kind;

then $10x$ = value of ten-dollar bills,

$5x$ = value of five-dollar bills,

$2x$ = value of two-dollar bills.

Hence $10x + 5x + 2x = 102$.

Adding terms, $17x = 102$;

dividing both members by 17, $x = 6$, number of bills of each kind.

VERIFICATION: $60 + 30 + 12 = 102$.

24. A newsboy, in counting his week's earnings, found that he had twice as many dimes as quarters, and 3 times as many nickels as dimes, and that he had in all \$7.50. How many pieces of money of each kind had he?

25. A newsboy has \$4.80 in quarters, dimes, and nickels, and of each an equal number. How many pieces of money has he?

26. A market woman has 4 times as many dimes as quarters, and twice as many nickels as dimes, and in all she has 52 pieces of money. How many pieces of each kind has she?

27. The difference between two numbers is 24, and the greater number is 4 times the less. What are the numbers?

Let $x =$ the less number;

then $4x =$ the greater number.

Hence $4x - x = 24$.

Combining terms, $3x = 24$;

whence $x = \frac{1}{3}$ of $24 = 8$, the less number,

and $4x = 4 \times 8 = 32$, the greater number.

28. A father's age is 3 times the age of his son, and the difference of their ages is 30 years. What is the age of each?

29. If a certain number is divided into two parts, their difference is 81, and the greater part is 4 times the less. What is the number?

30. A harness cost $12 more than a saddle, and the harness cost 3 times as much as the saddle. What was the cost of each?

31. A mother is 3 times as old as her daughter, and the difference of their ages is 30 years. What is the age of each?

32. A man is 9 times as old as his nephew, and the difference between their ages is 40 years. How old is each?

33. Four times a certain number is 45 more than the number. What is the number?

34. Ten times a certain number is 126 more than the number. What is the number?

35. A school enrolls 180 pupils, and there are 20 more girls than boys. How many pupils of each sex are in the school?

Let x = number of boys;
then $x + 20$ = number of girls.
Hence $x + x + 20 = 180$.
Combining terms, $2x + 20 = 180$;
subtracting 20 from each number, $2x = 160$;
whence $x = 80$, number of boys,
and $x + 20 = 80 + 20 = 100$, number of girls.

8. If a number $+ 5 = 35$, then the number $= 35 - 5$, which is 30. In like manner, if $x + 5 = 30$, then $x = 30 - 5 = 25$; and if $x - 5 = 25$, then $x = 25 + 5 = 30$. It is thus seen that *a number may be added to or subtracted from both members of an equation without affecting their equality.*

36. Cut a cord 60 feet long into two pieces such that one piece shall be 16 feet longer than the other.

37. A pole 120 feet long fell and broke into two pieces, one piece being 30 feet longer than the other. What was the length of each piece?

38. A is twice as old as B, and B is 15 years younger than C, and the sum of their ages is 95 years. How old is each?

39. Divide \$140 between two men, giving one \$20 more than the other.

40. The sum of two numbers is 120, and their difference is 20. What are the numbers?

41. A man bought a watch and chain for $85, and the cost of the watch was $5 more than 3 times the cost of the chain. What was the cost of each?

42. In a certain election 364 votes were polled by the two parties, and one party had 48 majority. How many votes were cast by each party?

43. In a certain village, containing 345 persons, there are 15 more women than men, and twice as many children as there are men and women together. How many of each are in the village?

44. Divide $1800 among three persons, giving to the second $200 more than to the first, and to the third $200 more than to the second.

45. The sum of two numbers is 50, and the greater is 2 more than twice the less. What are the numbers?

46. A tree 90 feet long was broken by the wind, and the part left standing was 20 feet shorter than the part broken off. What was the length of each part?

47. A and B are partners in business. A's capital is $500 less than twice B's, and their total capital is $5500. How much capital has each?

48. Divide a line 64 inches long into two parts such that the longer shall be 8 inches less than twice the shorter.

49. A school, enrolling 76 pupils, is divided into three classes. There are twice as many pupils in the second class as in the first, and 16 more in the third class than in the second. How many pupils are in each class?

50. A father is twice as old as his son, and the sum of their ages less 12 years is 60 years. How old is each?

51. A mother is 3 times as old as her daughter less 10 years, and the sum of their ages is 50 years. How old is each?

52. The sum of two numbers is 72, and the greater is 9 more than twice the less. What are the numbers?

53. A number multiplied by 8 exceeds 25 as much as 25 exceeds twice the original number. What is the number?

54. A father's age is now 4 times his son's age; but in 5 years his age will be only 3 times his son's age. How old is each?

55. A, B, and C together own 75 sheep. A has twice as many sheep as B, and C has 5 sheep less than A. How many sheep has each?

56. A man owns two farms which together contain 200 acres, and the larger farm contains 42 acres more than the smaller. How many acres are in each farm?

57. Twice A's age is 20 years more than B's age and 10 years more than C's age, and the sum of their ages is 120 years. What is the age of each?

58. A man owns two farms which together contain 180 acres of land; the first farm contains 30 acres less than twice the second. How many acres are in each farm?

59. A and B together have $180, and B has $20 less than 3 times A's money. How much money has each?

60. A lady bought 18 yards of silk and 15 yards of serge for $38.25, and the silk cost twice as much per yard as the serge. What was the cost of each?

61. A fruit dealer sold several dozen oranges at 25 cents a dozen, twice as many lemons at 15 cents a dozen, and twice as many pears as lemons at 10 cents a dozen. The bill for all was $3.80. How many dozen of each kind of fruit did he sell?

62. A man bought a suit of clothes for $30. The coat cost $5 more than the trousers, and the trousers twice as much as the vest. What was the cost of each garment?

63. A woman bought a cloak, a dress, and a bonnet for $35; the dress cost $8 more than the bonnet, and the cloak $4 more than the dress. What was the cost of each?

64. A jeweler sold three watches for $100. He sold the second watch for $15 more than the first, and the third watch for $5 less than the second. How much did he receive for each watch?

65. Two men start at the same time from two places which are 63 miles apart, and travel toward each other, one at the rate of 3 miles an hour, and the other at 4 miles an hour. In how many hours will they meet? How far will each travel?

SUGGESTION. — Let x = the number of hours.

NOTE. — For additional problems see pages 72 to 82.

EQUATIONS CONTAINING FRACTIONS

9. Since both members of an equation can be multiplied by the same number without affecting their equality (§ **5**), it is seen that an equation may be cleared of fractions by *multiplying both members by the L.C.M. of the denominators.*

1. Solve the equation $\frac{2}{3}x + \frac{3}{4}x = 17$.

Multiplying both members by 12 (L.C.M.), $8x + 9x = 204$;

adding terms, $17x = 204$;

dividing both members by 17, $x = 12$.

VERIFICATION: $8 + 9 = 17$.

Solve and verify the following equations:

2. $\frac{1}{2}x + \frac{2}{3}x = 21$.

3. $\frac{3}{4}x - \frac{2}{3}x = 2$.

4. $2x - \frac{5}{6}x = 14$.

5. $8 = \frac{1}{2}x + \frac{1}{6}x$.

6. $90 = 2x + \frac{1}{4}x$.

7. $\frac{4}{3}x - \frac{3}{4}x = \frac{3}{5}$.

8. $\frac{x}{2} + \frac{x}{5} - 2 = 5$.

9. $x + \frac{1}{2}x - \frac{1}{3}x = 14$.

10. $2x - \frac{3}{4}x + 5 = 15$.

11. $x - \frac{1}{3}x - \frac{1}{4}x = 15$.

12. $2x - \frac{2}{3}x - \frac{3}{4}x = 7$.

13. $\frac{1}{2}x + \frac{1}{3}x + \frac{1}{4}x - 5 = 8$.

14. $x + \frac{1}{2}x + \frac{3}{4}x - 6 = 12$.

15. $\frac{1}{2}x - \frac{1}{4}x + \frac{1}{3}x + 5 = 19$.

16. $x - \frac{1}{4}x - \frac{1}{3}x = 5$.

17. $\frac{x}{6} + \frac{x}{5} - \frac{3x}{10} + 6 = 10$.

18. $\frac{x+2}{3} + \frac{x-2}{4} = 2\frac{1}{2}$.

19. $\frac{x+2}{3} - \frac{x}{4} - \frac{5}{6} = 1\frac{1}{3}$.

20. $3(x-2) - \frac{3}{4}x = 7\frac{1}{2}$.

PROBLEMS

1. Divide 58 into three parts, such that the second part shall be $\frac{2}{3}$ of the first, and the third $\frac{3}{4}$ of the first.

Let x = first part;

then $\frac{2}{3}x$ = second part,

and $\frac{3}{4}x$ = third part.

Hence $x + \frac{2}{3}x + \frac{3}{4}x = 58$.

Multiplying by 12 (L.C.M.), $12x + 8x + 9x = 696$;

adding terms, $29x = 696$;

dividing by 29, $x = 24$, first part;

$\frac{2}{3}x = 16$, second part;

$\frac{3}{4}x = 18$, third part.

VERIFICATION: $24 + 16 + 18 = 58$.

2. Divide 48 into two parts, such that $\frac{2}{5}$ of the first part is equal to $\frac{2}{3}$ of the second.

3. Divide 36 into three parts, such that the second part is $\frac{2}{3}$ of the first, and the third $\frac{1}{2}$ of the second.

4. The difference between $\frac{7}{8}$ and $\frac{5}{6}$ of a number is 12. What is the number?

5. If to a man's age its half, its third, and 28 years are added, the sum will be 3 times his age. What is his age?

6. A boy being asked his age, replied that $\frac{2}{3}$ of his age was 4 years more than $\frac{2}{5}$ of his age. What was his age?

7. A boy gave $\frac{3}{8}$ of his money for a knife, and $\frac{1}{3}$ of it for a slate, and then had left 7 cents. How much money had he at first?

8. A farmer sold $\frac{2}{3}$ of his sheep to A, and $\frac{1}{4}$ of the remainder to B, and then had 30 sheep. How many sheep had he at first?

9. A farmer sold $\frac{3}{5}$ of his sheep, then bought $\frac{1}{2}$ as many sheep as he had left, and then had 48 sheep. How many sheep had he at first?

10. A piece of flannel lost $\frac{2}{5}$ of its length in fulling by shrinkage, and then measured 36 yards. What was the length of the piece before fulling?

11. A horse and a cow cost \$90, and $\frac{6}{7}$ of the cost of the horse was 3 times the cost of the cow. What was the cost of each?

12. The perimeter of a rectangular field is 80 rods, and the width of the field is $\frac{1}{3}$ of its length. What are the dimensions of the field?

13. A man walked $\frac{1}{3}$ of his journey the first day, $\frac{2}{5}$ of it the second day, and 20 miles the third day. How many miles were in the journey?

14. A man bought a horse and carriage for \$250, and $\frac{1}{3}$ the cost of the carriage was \$25 more than $\frac{1}{4}$ of the cost of the horse. What was the cost of each?

15. Divide \$240 among three persons, giving to the first $\frac{3}{5}$ as much as to the second, and to the third $\frac{1}{2}$ as much as to the first and second together.

16. A's age is $\frac{3}{4}$ of B's age, and 15 years ago A's age was $\frac{3}{5}$ of B's age. What is the age of each?

17. At the time of their marriage 12 years ago, a wife's age was $\frac{3}{5}$ of her husband's age, but now her age is $\frac{5}{7}$ of her husband's age. How old is each?

18. The sum of the ages of A, B, and C is 65 years, and A's age is $\frac{2}{5}$ of B's, and B's is $\frac{5}{6}$ of C's. What is the age of each?

19. A and B together own 144 acres of land, and A owns $1\frac{2}{3}$ times as many acres as B. How many acres has each?

20. The sum of two numbers is 70, and $\frac{1}{4}$ of the greater is $\frac{1}{3}$ of the less. Find the numbers.

21. The difference of two numbers is 15, and $\frac{1}{5}$ of the greater is $\frac{2}{5}$ of the less. Find the numbers.

22. The sum of $\frac{1}{2}$, $\frac{1}{6}$, and $\frac{1}{12}$ of a number is 5 less than the number. What is the number?

23. A man bought a watch and chain for \$75, and $\frac{3}{4}$ the cost of the watch was 3 times the cost of the chain. What was the cost of each?

24. A man spent $\frac{1}{2}$ of his yearly income for house rent, $\frac{1}{2}$ of the remainder for provisions, and $\frac{1}{2}$ of what then remained for other expenses, and laid up \$20. What was his yearly income?

25. A, B, and C entered into a business partnership with a capital of \$7200. A furnished $\frac{1}{3}$ as much capital as B, and C furnished $\frac{1}{2}$ as much as A and B together. How much capital did each furnish?

26. A man left $\frac{1}{2}$ of his estate to his wife, $\frac{1}{3}$ of it to his son, and the remainder, which was \$1500, to an invalid sister. How much did each receive? What was the value of the estate?

27. A man owns three farms which together contain 151 acres. The first farm contains 21 acres more than the second, and the second contains $\frac{3}{4}$ as many acres as the third. How many acres does each farm contain?

28. A farmer sold $\frac{1}{2}$ of his wheat to A and $\frac{1}{3}$ of the wheat remaining to B, and then had 80 bushels. How many bushels of wheat had he at first?

29. A flag pole 130 feet high was broken by the wind, and $\frac{3}{5}$ of the part broken off was equal to $\frac{3}{8}$ of the part left standing. What was the length of each part?

30. A sold a horse for $\frac{1}{5}$ more than it cost him, and the buyer sold it for \$80, losing $\frac{1}{6}$ of what it cost him. How much did A pay for the horse?

31. Divide a sum of money among three persons, giving the first \$10 more than $\frac{1}{3}$ of it, the second \$15 more than $\frac{1}{5}$ of it, and the third the \$10 which remains. What is the sum of money divided?

32. Find two consecutive numbers such that $\frac{1}{5}$ of the greater is 3 more than $\frac{1}{7}$ of the less.

SUGGESTION. — Let x = the less number; then $x + 1$ = the greater.

33. A, B, and C together have a certain sum of money; A has $\frac{1}{2}$ of it less \$5, B $\frac{1}{3}$ of it plus \$5, and C has \$25. How much money have A and B?

34. The difference of two numbers is 20, and $\frac{1}{2}$ of one number plus 15 is $\frac{3}{5}$ of the other number. Find the numbers.

35. A, B, and C together have 340 shares of stock; $\frac{2}{5}$ of A's number of shares is equal to B's, and $\frac{3}{4}$ of B's is equal to C's. How many shares of stock has each?

36. Ten years ago a son's age was $\frac{1}{5}$ of his father's age, but now his age is $\frac{2}{5}$ of his father's age. What is the age of each?

37. A man, being asked his age, replied that $\frac{3}{5}$ of his age 10 years ago is $\frac{2}{5}$ of his age 10 years hence. What was his age?

38. A fruit dealer sold a certain number of bushels of apples at 75 cents a bushel; $\frac{2}{5}$ as many bushels of peaches at \$1.20 a bushel; and $\frac{3}{5}$ as many bushels of pears at 80 cents a bushel; and received for all \$42.75. How many bushels of each did he sell?

39. Three bins hold together 117 bushels of wheat; the second bin holds $\frac{1}{3}$ more bushels than the first, and the third $\frac{1}{2}$ as many bushels as the second. How many bushels does each bin hold?

40. A woman bought a dollar's worth of postage stamps. She bought $\frac{1}{2}$ as many five-cent stamps as two-cent stamps, and as many one-cent stamps as five-cent stamps. How many stamps of each kind did she buy?

41. A can do a piece of work in 4 days and B in 5 days. In how many days can both together do the work?

Let x = number of days in which both together can do the work;

then $\frac{1}{x}$ = part of work both can do in one day,

$\frac{1}{4}$ = part of work A can do in one day,

$\frac{1}{5}$ = part of work B can do in one day.

Hence $\frac{1}{4}+\frac{1}{5}=\frac{1}{x}$.

Clearing of fractions, $5x+4x=20$;

adding terms, $9x=20$;

dividing by 9, $x=2\frac{2}{9}$.

Hence A and B together can do the work in $2\frac{2}{9}$ days.

42. A can cut a field of oats in 6 days, and B in 8 days. In how many days can both together cut it?

43. A man can dig a ditch in 5 days, and his son can dig it in 10 days. In how many days can both together dig it?

44. A can do a piece of work in 4 days, B in 6 days, and C in 8 days. In how many days can all together do it?

45. A can build a wall in 9 days, B in 6 days, and C in 12 days. In how many days can all together build it?

46. A can cut an acre of wheat in 8 hours, and B in 12 hours. In how many hours can both together cut an acre?

47. A can build a house in 20 days, and A and B working together can build it in 12 days. In how many days can B alone build the house?

Let x = number of days in which B can build it;

then $\frac{1}{x}$ = part B can do in one day.

Hence $\frac{1}{12}-\frac{1}{20}=\frac{1}{x}$.

Clearing of fractions, $5x-3x=60$;

combining terms, $2x=60$;

whence $x=30$, number of days in which B can build it.

48. A and B, working together, can build a wall in 8 days, and B alone can build it in 10 days. In how many days can A build it?

49. A man can do a piece of work in 9 days, and with his son's assistance he can do it in 6 days. In how many days can the son do the work?

50. A and B together can dig a ditch in 12 days, and B alone in 20 days. If B stops work when one half of the ditch is dug, how long will it take A to complete the work?

51. If 15 men can do a piece of work in 12 days, how many men can do the same work in 9 days? In 6 days?

52. A can mow an acre of grass in $\frac{3}{4}$ of a day, and B in $\frac{3}{5}$ of a day. How long will it take both together to mow 9 acres?

53. A cistern holding 140 gallons is filled from three pipes. The second discharges $\frac{1}{2}$ as much as the first and twice as much as the third. How many gallons are discharged by each pipe?

54. The sum of three numbers is 36, and the first is $\frac{1}{2}$ of the second, and the second $\frac{1}{3}$ of the third. What are the numbers?

55. A man owning $\frac{2}{3}$ of a factory sold $\frac{2}{5}$ of his share for $2000. At this rate what was the value of the factory?

56. A grammar school enrolls 250 pupils in three grades. The number of pupils in the first grade is $\frac{2}{3}$ of the number in the second grade and $\frac{3}{5}$ of the number in the third grade. How many pupils are in each grade?

57. A person being asked the time of day replied that $\frac{1}{2}$ the time past noon was equal to the time to midnight. What was the time?

58. The perimeter of a rectangle is 40 inches, and its width is $\frac{2}{3}$ of its length. What are its dimensions?

59. The difference between $\frac{3}{4}$ and $\frac{2}{3}$ of a number is 3. What is the number?

60. The sum of $\frac{1}{6}$ and $\frac{1}{9}$ of a number is 15. What is the number?

61. The sum of A's, B's, and C's ages is 70 years, and A's age is twice B's and $\frac{1}{2}$ of C's. What is the age of each?

62. A fishing rod consists of two parts. The length of the upper part is $2\frac{1}{2}$ times that of the lower part, and the difference in their lengths is 33 inches. What is the length of the rod?

63. Divide the number 45 into two parts such that the less shall be $\frac{4}{5}$ of the greater.

64. Find a number such that the sum of $\frac{1}{3}$ and $\frac{1}{2}$ of it is 10 more than $\frac{1}{6}$ of it.

65. An estate was divided among a widow, a son, and a daughter, the widow receiving $\frac{1}{3}$ of it, the son $\frac{1}{4}$ of it, and the daughter \$3250. What was the value of the estate?

66. A father bequeathed \$17000 to two sons, giving the younger $\frac{7}{10}$ as much as the elder. What was each son's share?

67. A man paid \$8100 for two farms, and $\frac{3}{5}$ of the cost of the larger farm was equal to $\frac{9}{10}$ of the cost of the smaller. What was the cost of each farm?

68. One fifth of a pole is in the ground, $\frac{1}{4}$ of it in the water, and 11 feet of it above the water. How long is the pole?

69. Two ninths of a pole is in the ground, $\frac{3}{7}$ of it in the water, and 21 feet in the air. How long is the pole?

70. When Charles is $\frac{2}{5}$ older than he now is he will be 21 years old. How old is Charles?

71. A man is 45 years old, and $\frac{2}{5}$ of his age is $\frac{3}{7}$ of the age of his wife. How old is his wife?

72. A man bought a farm, paying $\frac{2}{5}$ of the price down, $\frac{1}{3}$ of it the first year, $\frac{1}{6}$ of it the second year, and the remainder, which was \$600, the third year. What was the cost of the farm?

73. A flagstaff 80 feet long consists of two parts, and the shorter part is $\frac{3}{5}$ of the longer. How long is each part?

74. A horse and carriage cost \$350, and the horse cost $\frac{3}{4}$ as much as the carriage. What was the cost of each?

75. A man who earns \$60 a month saves each month $\frac{1}{5}$ as much as he spends. How much does he save?

76. A and B are partners in a business that yields a profit of \$4800, and A's share of the profit is $\frac{3}{5}$ of B's share. What is the share of each?

To Teachers. — If the time to be devoted to algebra is limited, omit as many of the exercises on pages 32–63 as may be necessary to provide time for the completion of the book. The most important algebraic work for grammar school pupils is the solution of problems.

ADDITION

10. A number preceded by the sign + is **positive**, and a number preceded by the sign − is **negative**. Thus, $+3$ or $+a$ is positive, and -3 or $-a$ is negative.

When no sign is written before a number, the sign + is understood. Thus, $3a$ is equivalent to $+3a$.

11. Two or more algebraic numbers are **like** when their literal parts are the same. Thus, $2ab$ and $5ab$ are like numbers; also $4a^2b$ and $-5a^2b$.

Like numbers have the same letters, and each letter the same exponent. $2a^2b^2$ and $3ab$ are not like numbers. Like numbers are also called **similar**.

12. To add like numbers with the same sign.

The sum of $3a$, $5a$, and $9a$ is $17a$, and the sum of $-3a$, $-5a$, and $-9a$ is $-17a$.

Add:

1. $5ac$, $6ac$, ac.
2. $-3bc$, $-2bc$, $-4bc$.
3. $12abc$, $5abc$, abc.
4. $-3ax$, $-ax$, $-5ax$.
5. x^2y, $7x^2y$, $5x^2y$.
6. $-4a^2y$, $-3a^2y$, $-a^2y$.
7. $-3x^2$, $-2x^2$, $-5x^2$.
8. ax, $2ax$, ax, $7ax$.

The sum of like numbers with the same sign is found *by adding the coefficients, annexing the literal part to their sum, and prefixing the common sign.*

13. To add like numbers with unlike signs.

9. What is the sum of $5\,ax$, $-3\,ax$, $6\,ax$, $-4\,ax$, and ax?

The sum of $5\,ax$, $6\,ax$, and ax is $12\,ax$; the sum of $-3\,ax$ and $-4\,ax$ is $-7\,ax$; $12\,ax - 7\,ax$ is $5\,ax$, the sum. If preferred, the numbers may be written in a column before adding.

Add:

10. $7\,ab$, $-2\,ab$, $-5\,ab$, $6\,ab$, $3\,ab$.

11. $-ac$, $-3\,ac$, $5\,ac$, $-6\,ac$, $7\,ac$, $4\,ac$.

12. $5\,ax$, $-4\,ax$, $6\,ax$, $-3\,ax$, $-9\,ax$.

13. $3\,xy$, $5\,xy$, $-xy$, $-7\,xy$, $-4\,xy$.

14. $-4\,x^2y$, $-6\,x^2y$, x^2y, $7\,x^2y$, $-3\,x^2y$.

Like numbers with unlike signs are added *by adding the positive numbers and the negative numbers and prefixing to the arithmetical difference of their sums the sign of the greater.*

14. To add unlike numbers.

In arithmetic the expression $5 + 7$ indicates that the two numbers are to be added, and the result (12) is their sum; but in algebra the expression $a + b$ indicates not only that a and b are to be added but also their sum. Thus, the sum of a and c is $a + c$; the sum of a and $-c$ is $a - c$; the sum of a, b, and $-c$ is $a + b - c$; and the sum of $3\,a$, a, and $-c$ is $4\,a - c$.

Find the sum of:

15. $a,\ b,\ c,\ d.$

16. $a,\ -b,\ c.$

17. $3a,\ 7a,\ -b,\ -2c.$

18. $12x^2y,\ 5x^2y,\ -x^2y.$

19. $5a^2b,\ -10ab,\ -3a^2b.$

20. $2a^2,\ 5ab,\ -a^2,\ -b^2.$

21. $5a^2b,\ -3ab^2,\ 6ab^2,\ -3a^2b,\ 2a^2b.$

22. $7x^2y^2,\ -6xy,\ 3xy,\ -4x^2y^2,\ -x^2y^2.$

23. $3a^2bc,\ -3c,\ 3b,\ -2a^2bc,\ 3c.$

15. To add polynomials.

An algebraic expression whose parts are not separated by the sign + or − is called a **term.** Thus, $3a^2bc$ is a term. An algebraic expression consisting of only one term, as $3abc$, is a **monomial;** one consisting of two terms, as $ab+c$, is a **binomial;** of three terms, as a^2+ab+b^2, a **trinomial;** and of two or more terms, as a^2+b^2, or a^2-bc+b^2-c, a **polynomial.**

24. Add the polynomials, $5a+b$, $12a-10b$, $3b-10a-5c$, and $a-5b+7c$.

PROCESS

$$\begin{array}{r} 5a+b \\ 12a-10b \\ -10a+3b-5c \\ a-5b+7c \\ \hline 8a-11b+2c \end{array}$$

Write the like terms in columns, as at the left, and then add as in § 14. The terms of a polynomial may be arranged in any order.

25. Add $18ab+9bc-15ac$, $13ac-12ab-7bc$, $8ab+14bc-5ac$, $3ab-12ac-11bc$.

26. $7\,ab - 5\,b + 6\,c$, $6\,b - 5\,ab - 3\,c$, $4\,ab - 12\,c + 3\,b$.

27. $9\,ax^2 - 6\,ax + 5\,a^2x$, $7\,ax - 15\,ax^2 - 15\,a^2x$, $10\,a^2x + 8\,ax^2 - 16\,ax$.

28. $8\,a^2b^2x^2 - 3\,abc + 2\,xy$, $5\,a^2b^2x^2 - 5\,xy + 4\,abc$, $4\,xy - 2\,abc - 7\,a^2b^2x^2$.

29. $7\,a^2m^2n - 5\,axy^2 + 6\,xy$, $4\,axy^2 - 5\,a^2m^2n^2 + 4\,xy$, $axy^2 + 3\,a^2m^2n - 7\,xy$.

30. $13\,mnx^2 - 7\,mx^2 + 4\,mnx$, $6\,mnx^2 + 5\,mx^2 - 3\,mnx$, $10\,mx^2 - 9\,mnx^2 - mnx$.

31. $9\,a^2b^2 - 3\,ab + 7\,ac$, $5\,ab - 7\,a^2b^2$, $4\,a^2b^2 + 2\,ab - 3\,ac$.

32. $6\,a^2x^2 - 5\,ab + 3\,abx$, $4\,abx - 2\,ab + 4\,a^2x^2$, $7\,ab - 6\,abx - 7\,a^2x^2$.

33. $6(a+b) - 7(a+b) - 3(a+b) + 8(a+b)$.

34. $10(x+y) - 8(x+y) + 5(x+y) - 3(x-y)$.

35. Reduce to simplest form $a^2b - 7\,ax^2 + 4\,ac - 3\,a^2b + 6\,ax^2 - 4\,ac + 2\,a^2b$.

36. Reduce to simplest form $4\,ax + 2\,b^2 - 3\,ac + 5\,b^2 - 10\,ax + 5\,ac - 2\,ax - 6\,b^2 + 12\,ax - ac$.

To TEACHERS. — Give pupils additional exercises when these may be needed to secure desired accuracy and skill.

SUBTRACTION

16. The difference of two given numbers is found by subtracting the second from the first, the number first given being the minuend.

1. From $7\,a$ take $4\,a$; from $7\,a^2b$ take $4\,a^2b$.

$$7\,a - 4\,a = 3\,a;\ 7\,a^2b - 4\,a^2b = 3\,a^2b.$$

2. From $7\,a$ take $-4\,a$; from $7\,a^2b$ take $-4\,a^2b$.

$$7\,a - (-4\,a) = 7\,a + 4\,a = 11\,a;$$
$$7\,a^2b - (-4\,a^2b) = 7\,a^2b + 4\,a^2b = 11\,a^2b.$$

It is thus seen that a positive monomial is subtracted by changing its sign to $-$, and a negative monomial by changing its sign to $+$, and then combining the resulting numbers as in addition.

The difference of a positive number and a negative number is their arithmetical sum. This may be illustrated by the teacher by assuming that in distance $+$ means to the right and $-$ to the left.

$$+7 - (+4) = 3;\ +7 - (-4) = 11.$$

NOTE. — This principle may also be illustrated by the use of the thermometer, the degrees above zero being considered positive, and those below zero negative. Thus, $+8°$ denotes $8°$ above zero, and $-12°$ denotes $12°$ below zero, and the difference between $+8°$ and $-12°$ is $20°$.

3. From a take b; from a take $-b$.

4. From $2a$ take 5; from $3a$ take -4.

5. From $-8a$ take $3a$; from $8a$ take $-3a$.

6. From $7ax$ take $-5ax$; from $-7ax$ take $-5ax$.

7. From $13xy$ take $-5xy$; from $-10xy$ take $6xy$.

8. From $-12x^2y$ take $8x^2y$; from $6x^2y$ take $-4x^2y$.

A monomial may be subtracted *by changing its sign and then proceeding as in addition.*

17. To subtract polynomials.

9. From $7a$ take $3a+b$.

$$7a-(3a+b)=7a-3a-b=4a-b.$$

Since the sum of $3a$ and b is to be taken from $7a$, both $3a$ and b must be subtracted.

10. From $7a$ take $3a-b$.

$$7a-(3a-b)=7a-3a+b=4a+b.$$

Since only the difference of $3a$ and b is to be taken from $7a$, if $3a$ be first subtracted the result will be too small by b, and so b must be added to the result.

11. From $7ax-3b^2x$ take $5ax-2b^2x+2c$.

PROCESS

$$\begin{array}{r} 7ax-3b^2x \\ \underline{5ax-2b^2x+2c} \\ 2ax-b^2x-2c \end{array}$$

Write the like terms in the same column. Change mentally the signs of $5ax$, $-2b^2x$, and $2c$, and proceed as in addition.

12. From $10a-5b+3c$ take $8a-2b+5c$.

13. From $15a^2-7ab$ take $10a^2+3ab-4c$.

14. From $a-b+c$ take $a-b+c-d$.

15. From $9x^2-5xy+4$ take $2xy-5x^2+6$.

16. From $a+b$ take $a-b$, and prove by addition.

17. From $5a^2b-3bc^2$ take $2a^2b-2bc^2$, and prove by addition.

18. From the sum of $x^2+2xy+y^2$ and x^2-y^2 take $x^2-2xy+y^2$.

19. From the sum of $3ax^2+5bx$ and $-5ax^2-4bx$ take ax^2-bx.

20. From the sum of $6a^2x^2-3ax$ and $-4a^2x^2+7ax$ take a^2x^2-6ax.

21. From the sum of $4x^2y-5ab$ and $7x^2y+12ab$ take the sum of $4x^2y-ab^2$ and $10x^2y+ab^2$.

A polynomial is subtracted *by changing the signs of the terms of the subtrahend, and then proceeding as in addition.*

18. Parentheses.

When a number included in a parenthesis is preceded by the sign $+$, the parenthesis may be removed without changing the signs of the terms within it. Thus, $3a+(4a-b)=3a+4a-b$.

But when a number included in a parenthesis is preceded by the sign $-$, the signs of the terms within the parenthesis must be changed ($+$ to $-$, and $-$ to $+$) if the parenthesis is removed. Thus, $6a-(4a-b)=6a-4a+b$.

The sign of $4a$ in the parenthesis is $+$, and the polynomial is the same as $6a-(+4a-b)$.

Remove the parenthesis, and reduce the polynomial to its simplest form:

22. $5x + (3x - 4)$.
23. $4x + (3 - 2x)$.
24. $3a - (a + 3)$.
25. $5a - (2a - 3)$.
26. $2x - (1 - 3x)$.
27. $x + 5 - (x - 5)$.
28. $\frac{1}{4}x - \frac{1}{2} - (3x - \frac{3}{4})$.
29. $ab + c - (ab + 2c)$.
30. $x^2 + 2xy - (x^2 - 2xy + y^2)$.
31. $a^2 - b^2 - (a^2 - ab - b^2)$.

19. The terms of a polynomial which follow the sign + may be inclosed in a parenthesis without changing the signs of the inclosed terms. Thus, $a + b - c + d$ may be changed to $a + (b - c + d)$.

The terms of a polynomial which follow the sign − may be inclosed in a parenthesis if the sign of each inclosed term be changed. Thus, $a - b + c - d$ may be changed to $a - (b - c + d)$.

Put in a parenthesis:

32. The last three terms in $5a^2 - 2b^2 - 3ab + c^2$.
33. The second and third terms in $x^2 - 2xy + y^2 - 5$.
34. The last three terms in $ab - a^2 + 2ab - b^2$.
35. The last two terms in $a^3 - 3a^2b - 3ab^2 + b^3$.

EQUATIONS AND PROBLEMS

Find the value of x in the following equations:

36. $5x - (3x - 2) = 18$.
37. $7x - (5 + 3x) = 15$.
38. $9x + (6 - 4x) = 26$.
39. $7x - (3x + 12) = 28$.

Find the value of x:

40. $2x-(4-2x)=80.$

41. $6x-(x+6)=44.$

42. $7x-(3x-4)=32.$

43. $12x-(10+7x)=20.$

44. $10x-(12-4x)=30.$

45. $8x+(-3x-6)=24.$

46. $-5x\times(-\frac{3}{5})=21.$

47. $6x-(-4x+8)=12.$

48. $\frac{x}{4}-\frac{6-3x}{8}=18.$

49. $3x-\frac{x+6}{2}=22.$

50. $\frac{6x}{4}-\frac{4x+4}{3}=2.$

51. $4x+\frac{2x-4}{4}=8.$

52. $\frac{3x-1}{3}-\frac{x-1}{2}=3\frac{2}{3}.$

53. $3x-(5-2x)=12-(2x-4).$

54. $10x+(-4x+10)=5x-(3x-14).$

55. If a number less 12 is subtracted from 3 times the number, the difference will be 42. What is the number?

Let x = the number;

then $3x-(x-12)=42.$

Removing parenthesis, $3x-x+12=42;$

whence $2x=30;$

$x=15$, the number.

56. If a number plus 12 is subtracted from 3 times the number, the difference will be 18. What is the number?

57. If twice a number less 10 is subtracted from 5 times the number, the difference will be 40. What is the number?

58. If twice a number plus 10 is subtracted from 5 times the number, the difference will be 50. What is the number?

MULTIPLICATION

20. A number written at the right and above another number, as the ^{4} in 7^4, is called an **exponent.** It shows how many times the number is taken as a factor. Thus, $5^3 = 5 \times 5 \times 5$, and $a^3 = a \times a \times a$.

When an exponent is a figure, as in a^5, it is called a **numerical** exponent, and when it is a letter, as in a^m, it is called a **literal** exponent. When a number has no exponent expressed, the exponent ^{1} is understood. Thus, $6 = 6^1$ and $x = x^1$.

21. The exponent of any letter in a product is equal *to the sum of its exponents in the factors.* Thus, $a^3 \times a^2 = a^{3+2} = a^5$, and $a^m \times a^n = a^{m+n}$. Hence the product of $5a^2b^2c$ and a^3bc^2 is $5a^5b^3c^3$.

Multiply:

1. $3a^2bx^2$ by $5a^2b^4x$.
2. $4x^3y^2z$ by $6xy^3z^3$.
3. $6a^2x^2y$ by $3axy^2$.
4. $7a^2b^2c^2$ by ab^3c.
5. $3ab^2x^3$ by $5a^3bx^2$.
6. $10a^4b^2x$ by $7ab^2x^3$.

22. $a \times b$ or $-a \times (-b) = +ab$, and $-a \times b$ or $a \times (-b) = -ab$. It is thus seen that when two factors have *like* signs, their product is *positive*, and when two factors have *unlike* signs, their product is *negative;* or,

briefly stated, in multiplication *like signs give* + *and unlike signs* −.

7. What is the product of -3×4? $5 \times (-3)$? $\frac{1}{2} \times (-10)$? $-\frac{1}{2} \times (-5)$? $-a \times b$? $-a \times (-b)$? $a \times (-b)$? $ab \times (-a)$? $-ab \times (-b)$?

Multiply:

8. $-2a^2bx$ by $-6ab^2x^2$.
9. $-4ab^2x^4$ by $3a^2bx$.
10. $9ax^3y^2$ by $-4a^3bx$.
11. $-7ab^2c$ by $-5a^2bc^2$.
12. $-3x^2y$ by $-4xy^2$.
13. $-4a^2bx^2$ by $\frac{1}{2}a^3b^2x$.
14. $\frac{3}{4}a^3x^2y^3$ by $-8ax^2y^2$.
15. $-6axy^3$ by $-\frac{1}{2}a^2x^2y$.

The product of two or more monomials is found *by multiplying together the coefficients, prefixing the proper sign, and then annexing all the letters each with an exponent equal to the sum of its exponents in the several factors.*

23. A polynomial may be multiplied by a monomial *by multiplying each term of the polynomial by the monomial.*

16. Multiply $3a^2 - 5ab + a^3c$ by $-4ac$.

PROCESS

$$\begin{array}{l} 3a^2 - 5ab + a^3c \\ -4ac \\ \hline -12a^3c + 20a^2bc - 4a^4c^2 \end{array}$$

Multiply each term of the multiplicand by $-4ac$.

Multiply:

17. $a^2 - 2ab + b^2$ by $-4ab$.

18. $3x^3 - 2x^2y + 5xy^2 + y^3$ by $4xy^2$.

19. $12y^3 - 4xy + 2x^2 + x^3$ by $-3x^2y$.

20. $3x + 15x^2y^2 - 6y^3$ by $-\frac{1}{3}xy$; by $-x^2y^2$.

21. $a^2 - 2ab + b^2$ by $-3a^2b$; by $-2ab^2$.

22. $8x^2y - 12xy^2 - 4y^3$ by $-\frac{1}{2}xy^2$; by $\frac{3}{4}x^2y$.

23. $x^3 - 2x^2 + 5x - 4$ by $2xy$; by $-5x^2y^2$.

24. $3x^2 + xy - 6$ by $-3y^2$; by $-4x^2y^2$.

25. $4y^2 - 4xy + 2x^2$ by $-5xy^2$; by $\frac{1}{2}xy^2$.

24. When the multiplier in arithmetic contains two or more orders, as 45, the product is found by multiplying the multiplicand successively by the number denoted by each figure in the multiplier, and then adding the partial products.

In like manner in algebra when the multiplier is a polynomial, the product is found *by multiplying the multiplicand by each term of the multiplier and adding the partial products*, as shown below.

26. Multiply $a^2 - 2ab + b^2$ by $a - 2b$.

PROCESS

$$\begin{array}{l} a^2 - 2ab + b^2 \\ a - 2b \\ \hline a^3 - 2a^2b + ab^2 \\ \quad - 2a^2b + 4ab^2 - 2b^3 \\ \hline a^3 - 4a^2b + 5ab^2 - 2b^3 \end{array}$$

Write the factors as at the left. Multiply by a and then by $-2b$, and add the two products.

The terms of each polynomial should be arranged according to the ascending or descending powers of the same letter. In multiplying observe carefully the signs.

Multiply:

27. $a^2 - 2ab + b^2$ by $a - b$; by $a^2 - b^2$.

28. $x^3 - 3x^2 + 5$ by $x^3 - x^2 - 3$.

29. $3a^2 - 4ab + 2b^2$ by $a^2 - 2ab$.

30. $x^2 - 2xy + y^2$ by $x^2 + 2xy + y^2$.

31. $x^3 - 4x^2 + 11x - 24$ by $x^2 + 4x + 5$.

32. $3x + x^2 + 9$ by $9 - 3x + x^2$.

Arrange the terms thus: $x^2 + 3x + 9$ by $x^2 - 3x + 9$.

33. $10 + x^3 + 2x + 6x^2$ by $x^3 - 6x^2 + 2$.

34. $x^3 - 4x^2 + 11x - 24$ by $x^2 - x$.

35. $1 + x + x^2$ by $1 - x^2$.

36. $x + y + 1$ by $x - y - 1$.

37. $x^3 - 3x^2y + 3xy^2 - y^3$ by $x - y$.

38. $a^2 + 2ab + b^2$ by $a + b$.

39. $a + b + 1$ by $a - b + 1$.

40. $1 + 2x + x^2$ by $1 + x$.

41. $a^3 - b^3$ by $a^2 - b^2$.

42. $2ab - 3c$ by $2ab + 3c$.

43. $2a - 2b$ by $2a + 2b$.

44. $5a + 3x$ by $a - x$.

45. $3x + y$ by $3x - 2$.

DIVISION

25. Division is the inverse of multiplication. In multiplication two factors are given to find their product; in division a product and one of its factors are given to find the other factor.

26. In multiplying monomials "*like signs give* + *and unlike signs give* − " (§ 22), and the same law is true in the division of one monomial by another. Thus, $ab \div b = a$ and $-ab \div (-b) = a$; $ab \div (-b) = -a$, and $-ab \div b = -a$.

1. What is the quotient of $12 \div (-3)$? $-15 \div 5$? $-21 \div (-7)$? $ab \div a$? $-ab \div a$? $-ab \div (-a)$? $-a^2b \div (-ab)$? $-6x^2y^2 \div 3x^2y$?

In multiplying monomials the product of the coefficients is the coefficient of the product, and hence in dividing one monomial by another, *the coefficient of the dividend is divided by the coefficient of the divisor.* Thus, $12a \div 3a = 4$.

In multiplying monomials the exponents of each letter are added (§ 21), and hence in dividing one monomial by another *the exponent of each letter in the divisor is subtracted from the exponent of the same letter in the dividend.* Thus,

$$12a^5b^3 \div 3a^3b^2 = 4a^{5-3}b^{3-2} = 4a^2b.$$

2. Divide $21\,a^4x^2$ by $-7\,a^2x$.

PROCESS: $\dfrac{21\,a^4x^2}{-7\,a^2x} = -3\,a^2x.$

Divide:

3. $-28\,x^5y^3$ by $4\,x^2y$.

4. $56\,a^6b^5c^2$ by $-7\,a^2b^2c^2$

5. $-36\,b^5c^4d$ by $12\,b^3c^2d$.

6. $-63\,a^2x^2y^3$ by $-9\,x^2y^3$.

7. $24\,a^5bcx^2$ by $-12\,a^4bx^2$.

8. $-56\,x^7y^5z$ by $-8\,x^5y^3z$.

9. $75\,a^5x^3y^2$ by $-25\,a^2x^2y^2$.

10. $-48\,x^3y^3z^2$ by $-8\,x^2y^2z$; by $6\,x^3y^2z$.

11. $-81\,a^3m^3n^3$ by $-9\,a^2m^3n^2$; by $27\,a^3mn$.

12. $-42\,a^4x^3y^3$ by $-14\,a^2x^3y^2$; by $7\,ax^2y^2$.

13. $56\,a^3b^3x^3$ by $-7\,a^2b^2x^3$; by $8\,abx$.

14. $-63\,m^4n^2x^4$ by $-9\,m^2n^2x^2$; by $21\,m^3nx^3$.

15. $-120\,a^4b^4x^4$ by $30\,a^2b^4x^2$; by $-15\,a^3b^3x^3$.

27. A polynomial is divided by a monomial *by dividing successively each term of the polynomial by the monomial.*

16. Divide $8\,x^2y - 16\,x^5y^2 - 4\,x^4$ by $4\,x^2$.

PROCESS: $\dfrac{8\,x^2y - 16\,x^5y^2 - 4\,x^4 \;\big|\; 4\,x^2}{2\,y - 4\,x^3y^2 - \quad x^2,}$ quotient.

Divide:

17. $12\,a^3x^4 - 8\,a^2x^3y^2 + 4\,a^4x^2y^3$ by $4\,a^2x^2$.

18. $25\,x^2yz - 20\,xy^2z + 15\,xyz^2$ by $-5\,xyz$.

19. $60\,a^3b^3c^3 - 48\,a^2b^4c^2 - 36\,a^2b^2c^2$ by $-12\,a^2b^2c^2$.

28. The first step in dividing one polynomial by another is to arrange the terms of each according to the ascending or descending powers of a common letter, as shown below.

20. Divide $4\,a^2b^4 + a^6 - b^6 - 4\,a^4b^2$ by $a^2 - b^2$.

$$
\begin{array}{l|l}
a^6 - 4\,a^4b^2 + 4\,a^2b^4 - b^6 & a^2 - b^2 \\
\underline{a^6 - a^4b^2} & \overline{a^4 - 3\,a^2b^2 + b^4}, \text{ quotient.} \\
-3\,a^4b^2 + 4\,a^2b^4 & \\
\underline{-3\,a^4b^2 + 3\,a^2b^4} & \\
a^2b^4 - b^6 & \\
\underline{a^2b^4 - b^6} &
\end{array}
$$

Divide:

21. $a^2 + 2\,ab + b^2$ by $a + b$.

22. $a^2 - 2\,ab + b^2$ by $a - b$.

23. $a^2 - b^2$ by $a + b$; by $a - b$.

24. $4\,a^2 - 4\,ab + b^2$ by $2\,a - b$.

25. $9\,x^2 + 12\,xy + 4\,y^2$ by $3\,x + 2\,y$.

26. $x^4 - y^4$ by $x^2 - y^2$; by $x^2 + y^2$.

27. $x^4 - 2\,x^2y^2 + y^4$ by $x^2 - y^2$.

28. $1 - 2\,x + 3\,x^2 - 2\,x^3$ by $1 - x + 2\,x^2$.

29. $1 + 2\,x^2 - x^3 + 2\,x^4$ by $1 - x + x^2$.

Divide:

30. $a^3 - 3a^2b + 3ab^2 - b^3$ by $a^2 - 2ab + b^2$.

31. $x^2 + 4x - 32$ by $x + 8$; by $x - 4$.

32. $x^3 + 2x^2y + 2xy^2 + y^3$ by $x + y$.

33. $4a^2 - 12ab + 9b^2$ by $2a - 3b$.

34. $1 - x - 3x^2 - x^5$ by $1 - 3x + 2x^2 - x^3$.

35. $x^6 + x^5y + x^4y^2 + x^2y^4 + xy^5 + y^6$ by $x^2 + xy + y^2$.

36. $a^6 - 3a^4b^2 + 3a^2b^4 - b^6$ by $a^4 - 2a^2b^2 + b^4$.

37. $a^4 + a^2b^2 + b^4$ by $a^2 - ab + b^2$.

Insert $+ 0$ for the terms of the dividend which are wanting, thus: $a^4 + 0 + a^2b^2 + 0 + b^4$.

38. $x^4 - 17x^2 + 16$ by $x^2 + 5x + 4$.

39. $x^3 + a^3$ by $x^2 - ax + a^2$.

40. $a^3 - b^3$ by $a^2 + ab + b^2$.

41. $14x^4 - 10x^3 - 41x^2 + 25x + 15$ by $7x^2 - 5x - 3$.

42. $15x^4 - 14x^3 + 25x^2 - 6x - 9$ by $3x^2 - x - 1$.

43. $x^4 + 2x^2 - x + 2$ by $x^2 - x + 1$.

44. $x^4 - 2x^2 + 8x - 3$ by $x^2 + 2x - 1$.

45. $a^2 - a^2b - 2ab + ab^2 + b^2$ by $a - ab - b$.

46. $a^2 + ab - ac - bc$ by $a + b$.

47. $x^4 + x^3 - 9x^2 - 16x - 4$ by $x^2 + 4x + 4$.

FACTORING

29. To factor monomials.

Since $6 = 2 \times 3$ and $a^3 = a \times a \times a$, the prime factors of $6a^3$ are 2, 3, a, a, a. In like manner any composite monomial may be resolved into its prime factors *by factoring the coefficient and taking each letter in the literal part as many times as there are units in its exponent.*

If the monomial is negative, one factor is negative. Thus, the prime factors of $-6a^3$ are -2, 3, a, a, a.

Resolve into prime factors:

1. $9a^2b^2$.
2. $12x^3y$.
3. $10a^3bx^2$.
4. $15m^3n^2$.
5. $18a^3bc$.
6. $25x^2y^2$.
7. $-6x^3y^2$.
8. $-14a^2b^3$.
9. $-8x^2y^3z$.

30. Since $(3a^3)^2 = 3a^3 \times 3a^3 = 9a^{3\times 2} = 9a^6$, $\sqrt{9a^6} = \sqrt{9} \times \sqrt{a^6} = 3 \times a^{6\div 2} = 3a^3$.

The square root of a monomial is found *by extracting the square root of the coefficient and dividing the exponent of each letter by* 2.

The square root of a number is one of its two equal factors.

$16a^4b^2 = 4a^2b \times 4a^2b$; hence, $\sqrt{16a^4b^2} = 4a^2b$.

Resolve into two equal factors:

10. $9a^4b^4$.
11. $25x^6y^2$.
12. $64a^4x^2$.
13. $16m^2n^2$.
14. $36a^2b^4c^2$.
15. $49a^6x^4y^2$.
16. $49a^2x^4y^2$.
17. $81m^2n^2x^4$.
18. $36a^2b^2x^2$.

31. To factor binomials.

Since $(a+b)(a-b)=a^2-b^2$, the product of the sum and the difference of two numbers is *the difference of their squares.*

Write the product of:

19. $(x+y)(x-y)$.
20. $(a-c)(a+c)$.
21. $(x+2)(x-2)$.
22. $(2-x)(2+x)$.
23. $(2x+y)(2x-y)$.
24. $(2x+3y)(2x-3y)$.
25. $(a+2b)(a-2b)$.
26. $(ax-3b)(ax+3b)$.

32. Since $a^2-b^2=(a+b)(a-b)$, the difference of two squares is equal to the product of the sum and difference of their square roots.

A binomial expressing the difference of two squares is resolved into two factors *by taking the sum of the roots of the two squares for one factor and the difference of their roots for the other factor.*

Thus, $x^4-4y^2=(x^2+2y)(x^2-2y)$.

Resolve into factors:

27. a^2-x^2.
28. x^2-4.
29. $4-y^2$.
30. $4x^2-y^2$.
31. $9x^2-4y^2$.
32. $\frac{1}{4}a^2-4b^2$.
33. $a^2x^2-9b^2$.
34. $16a^2x^4-9$.

35. Resolve $3a^3b^2 - 12ax^2$ into factors.

Observe that $3a$ is a factor of each term.

$3a^3b^2 - 12ax^2 = 3a(a^2b^2 - 4x^2) = 3a(ab + 2x)(ab - 2x)$.

The factors are $3a$, $ab + 2x$, and $ab - 2x$.

36. Resolve $8a^2x - 18x^3y^2$ into factors.

37. Resolve $3x^4y^2 - 27x^2y^4$ into factors.

38. Resolve $4x^3 - 9x$ into factors.

33. To factor trinomials which are squares.

$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$; hence the square of the *sum* of two numbers is *the square of the first, plus twice the product of the first multiplied by the second, plus the square of the second.* Thus, $(2x + y)^2 = 4x^2 + 4xy + y^2$.

Write the square of:

39. $x + y$.	**42.** $x + 3$.	**45.** $2x + y$.
40. $a + x$.	**43.** $5 + x$.	**46.** $ax + 3$.
41. $x + b$.	**44.** $3a + b$.	**47.** $3x + 2y$.

34. $(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$; hence the square of the *difference* of two numbers is *the square of the first, minus twice the product of the first multiplied by the second, plus the square of the second.* Thus, $(x - 2y)^2 = x^2 - 4xy + 4y^2$.

Write the square of:

48. $x - y$.	**51.** $5 - x$.	**54.** $x - 3y$.
49. $x - a$.	**52.** $a^2 - b^2$.	**55.** $ax - 4$.
50. $x - 3$.	**53.** $3a - b$.	**56.** $2a - 3b$.

35. A trinomial is a perfect square if two of its terms are perfect squares and the remaining term is twice the product of their square roots.

The square root of a trinomial which is a perfect square is found *by connecting the square roots of the terms which are squares with the sign of the remaining term.*

Thus, $\sqrt{x^2 - 4xy + 4y^2} = \sqrt{x^2} - \sqrt{4y^2} = x - 2y$.

The square root of a trinomial is one of its two equal factors.

57. Resolve $4x^2 - 4xy + y^2$ into its factors.

$\sqrt{4x^2} = 2x$; $\sqrt{y^2} = y$; $2x - y$, the square root.

Hence $4x^2 - 4xy + y^2 = (2x - y)(2x - y)$.

Resolve into factors:

58. $a^2 + 6ab + 9b^2$.

59. $a^4 - 2a^2b + b^2$.

60. $1 - 10ab^2 + 25a^2b^4$.

61. $4y^2 - 4y + 1$.

62. $25x^2 + 10xy^2 + y^4$.

63. $x^2y^2 - 10xy + 25$.

64. Resolve into factors $3x^3y + 6x^2y^2 + 3xy^3$.

Observe that $3xy$ is a factor of each term.

$3x^3y + 6x^2y^2 + 3xy^3 = 3xy(x^2 + 2xy + y^2) = 3xy(x + y)(x + y)$.

Resolve into factors:

65. $2x^3 - 8x^2 + 8x$.

66. $x^3 - 10x^2 + 25x$.

67. $5a^3 - 10a^2b + 5ab^2$.

68. $2a^4 + 4a^2c^2 + 2c^4$.

69. $3x^4 + 12x^2 + 12$.

70. $4 - 40ab + 100a^2b^2$.

36. To find the product of two binomials with a common term.

It may be shown* that the product of any two binomials with a common term is a trinomial made up of (1) *the square of the common term*, (2) *the product of the common term multiplied by the algebraic sum of the two unlike terms*, and (3) *the product of the unlike terms.*

(1) $(x+a)(x-b) = x^2 + ax - bx - ab = x^2 + (a-b)x - ab.$

(2) $(x+4)(x-3) = x^2 + (4-3)x - 12 = x^2 + x - 12.$

Write the product of:

1. $(x+4)(x+3).$
2. $(x+5)(x-3).$
3. $(x+2)(x-5).$
4. $(x-5)(x-4).$
5. $(x-7)(x+3).$
6. $(x+a)(x+b).$
7. $(x-a)(x+b).$
8. $(x-a)(x-b).$
9. $(x+3c)(x-2c).$
10. $(x-3b)(x+5b).$

37. It follows from § 36 that a trinomial which is the product of two binomials with a common term may be resolved into two binomial factors.

11. Resolve into factors $x^2 + 5x + 6.$

$\sqrt{x^2} = x$, first term.

$6 = 2 \times 3;\ 2 + 3 = 5.$

Hence $x^2 + 5x + 6 = (x+2)(x+3).$

*See White's *School Algebra*, p. 88.

12. Resolve into factors $x^2 - 4x - 21$.

$\sqrt{x^2} = x$, first term.

$-21 = -7 \times 3$; $-7 + 3 = -4$.

Hence $x^2 - 4x - 21 = (x - 7)(x + 3)$.

It will be observed that each of the trinomials given above may be readily resolved into factors *by inspection.*

Resolve into factors:

13. $x^2 + 8x + 16$.

14. $x^2 + 7x + 10$.

15. $x^2 - 3x - 18$.

16. $x^2 + 2x - 63$.

17. $x^2 - 3x - 70$.

18. $x^2 - 15x + 56$.

19. $x^2 + 3x - 28$.

20. $x^2 + 7x - 18$.

21. $x^2 + x - 42$.

22. $x^2 - 11x + 28$.

23. $x^2 + 7x + 12$.

24. $x^2 - 12x + 35$.

25. $x^2 + 5x - 24$.

26. $x^2 - 7x - 60$.

27. $x^2 - 4x - 45$.

28. $x^2 + 2x - 15$.

Resolve into factors the first member of each of the following equations:

29. $x^2 - 9x + 14 = 0$.

30. $x^2 - 4x - 5 = 0$.

31. $x^2 - 10x + 24 = 0$.

32. $x^2 - x - 6 = 0$.

33. $x^2 - 2x - 8 = 0$.

34. $x^2 - 3x - 28 = 0$.

35. $x^2 - 7x + 6 = 0$.

36. $x^2 - x - 12 = 0$.

37. $x^2 - 2x - 15 = 0$.

38. $x^2 - 10x + 21 = 0$.

39. $x^2 - 12x + 32 = 0$.

40. $x^2 - 15x + 56 = 0$.

41. $x^2 - 4x - 60 = 0$.

42. $x^2 + 3x - 10 = 0$.

MISCELLANEOUS EXERCISES

Resolve into factors:

1. $x^2 - 25$.
2. $16 - a^2$.
3. $1 + 6a + 9a^2$.
4. $x^2 - 2x + 1$.
5. $3x^3 - 3xy^2$.
6. $x^3 - 10x^2 + 25x$.
7. $2x^3 + 8x^2 + 8x$.
8. $9a^4 - 36$.
9. $a^4 - b^4$.
10. $4x^2 - 4x + 1$.
11. $a^4 - 2a^2b^2 + b^4$.
12. $a^2 - 4b^4$.
13. $x^2 + 8x + 16$.
14. $x^2 - 2abx + a^2b^2$.
15. $5a^2b - 20ab^2 + 20b^3$.
16. $x^2 - 5x - 14$.
17. $x^2 - 11x + 18$.
18. $x^2 + 6x + 5$.
19. $100 - 20x + x^2$.
20. $48 - 24y + 3y^2$.
21. $3x^2 - 6xy + 3y^2$.
22. $7x^2 - 63$.
23. $x^3 - 5x + 9$.
24. $9a^2 - 6ab + b^2$.
25. $x^2 - 16$.
26. $25 - x^2$.
27. $9x^2 - 25$.
28. $4x^2 - 9$.
29. $36 - 9a^2$.
30. $x^2 - 2xy + y^2$.
31. $1 - 8a + 16a^2$.
32. $4a^2 + 4ab + b^2$.
33. $a^4 - 4b^4$.
34. $5x^2 - 45$.
35. $3x^2 - 4xy + 3y^2$.
36. $8x^2 - 50$.
37. $5x^2 - 20xy + 20y^2$.
38. $5 - 20y + 20y^2$.
39. $x^2 - 2x - 15$.
40. $x^2 - 7x - 18$.
41. $3a^2 - 6a + 3$.
42. $x^2 + 5x + 6$.
43. $x^2 - 2x - 3$.
44. $x^2 - 5x - 60$.
45. $x^2 + 7x - 18$.
46. $x^2 - 5x - 24$.

FRACTIONS

38. In algebra a fraction is treated as an *indicated division*, the numerator being the dividend, and the denominator the divisor.

The changing of the sign of either term of a fraction changes the sign of the fraction.

The processes with fractions are the same in algebra as in arithmetic.

Reduce to lowest terms:

1. $\frac{3a^2b^3}{15ab^2}$.

2. $\frac{3ab}{3a^3b-6ab}$.

3. $\frac{a^2-b^2}{a^2+ab}$.

4. $\frac{a^2-2ab+b^2}{a^2-b^2}$.

5. $\frac{a^2-b^2}{2a^2-2ab}$.

6. $\frac{x^2-y^2}{x^2+2xy+y^2}$.

Reduce to an integral or a mixed number:

7. $\frac{ax^2-bx^2}{x^2}$.

8. $\frac{ab+x}{a}$.

9. $\frac{x^2-y^2}{x+y}$.

10. $\frac{x^2+2xy+y^2}{x+y}$.

11. $\frac{x^2+3x-28}{x-4}$.

12. $\frac{abc-cx^2}{abc}$.

13. Reduce $a-\frac{a^2}{a+b}$ to a fraction.

PROCESS: $a-\frac{a^2}{a+b}=\frac{a(a+b)-a^2}{a+b}=\frac{ab}{a+b}$.

Reduce to a fraction:

14. $3x + \frac{x}{x-3}$.

15. $1 - \frac{x-7}{x+7}$.

16. $a + \frac{a^2+x^2}{a+x}$.

17. $a + x - \frac{2ax}{a+x}$.

18. $a - b + \frac{a^2+b^2}{a+b}$.

19. $5a - \frac{3ax-a}{x-1}$.

20. Reduce $\frac{a}{a-b}$, $\frac{b}{a+b}$, and $\frac{a^2}{a^2-b^2}$ to equivalent fractions with a common denominator.

The L.C.M. of $a-b$, $a+b$, and a^2-b^2 is a^2-b^2.

$$\frac{a}{a-b} = \frac{a(a+b)}{(a-b)(a+b)} = \frac{a^2+ab}{a^2-b^2}.$$

$$\frac{b}{a+b} = \frac{b(a-b)}{(a+b)(a-b)} = \frac{ab-b^2}{a^2-b^2}.$$

$$\frac{a^2}{a^2-b^2} = \frac{a^2}{a^2-b^2}.$$

Reduce to equivalent fractions with the lowest common denominator:

21. $\frac{a}{bc}$, $\frac{b}{ac}$, $\frac{c}{ab}$.

22. $\frac{a}{3c}$, $\frac{b}{4ac}$, $\frac{c}{6a}$.

23. $\frac{3}{5a}$, $\frac{5}{2a^2}$, $\frac{4}{10a}$.

24. $\frac{a-b}{a+b}$, $\frac{a+b}{a-b}$.

25. $\frac{1}{x+1}$, $\frac{1}{x^2-1}$, $\frac{1}{x+1}$.

26. $\frac{a-b}{a}$, $\frac{a-b}{a+b}$, $\frac{a+b}{a}$.

39. The sum or the difference of algebraic fractions is found *by connecting them with the proper sign, and then reducing the result to its simplest form.*

27. What is the sum of $\frac{1}{a-b}$ and $\frac{1}{a+b}$? What is their difference?

(1) $\frac{1}{a-b}+\frac{1}{a+b}=\frac{a+b}{a^2-b^2}+\frac{a-b}{a^2-b^2}=\frac{a+b+a-b}{a^2-b^2}=\frac{2a}{a^2-b^2}$.

(2) $\frac{1}{a-b}-\frac{1}{a+b}=\frac{a+b}{a^2-b^2}-\frac{a-b}{a^2-b^2}=\frac{a+b-(a-b)}{a^2-b^2}=\frac{2b}{a^2-b^2}$.

Observe that in (2) the second fraction is preceded by $-$; hence the signs in the numerator must be changed.

Simplify:

28. $\frac{1}{x+y}+\frac{1}{x-y}$.

29. $\frac{x+y}{x-y}+\frac{x-y}{x+y}$.

30. $\frac{5}{x-3}-\frac{3}{x-2}$.

31. $\frac{1}{a+b}+\frac{2b}{a^2-b^2}$.

32. $\frac{a^2}{a-b}-\frac{a^3}{a^2-b^2}$

33. $\frac{x^2+5}{xy}-\frac{x+3}{y}$.

34. $\frac{a-x}{ax}+\frac{a+x^2}{ax^2}$.

35. $\frac{x-y}{x}-\frac{y-x}{y}$.

36. $\frac{5x}{x+4}+\frac{2}{x-3}$.

37. $\frac{x-2}{x+3}-\frac{x+4}{x-5}$.

38. $\frac{a-b}{a}-\frac{a}{a+b}$.

39. $\frac{1}{a-b}-\frac{1}{a+b}$.

40. $\frac{a^2}{a^2-b^2}-\frac{a}{a+b}$.

41. $\frac{1+2a}{1-2a}-\frac{1-2a}{1+2a}$.

42. $\frac{1}{x-a}-\frac{1}{x+a}+\frac{2x}{x^2-a^2}$.

43. $\frac{ab}{a+b}-\frac{ab}{a-b}+\frac{1}{a-b}$.

44. $\frac{1}{1+x}+\frac{1}{1-x}+\frac{2x}{1-x^2}$.

45. $\frac{2a}{a^2-1}+\frac{5}{a-1}-\frac{3}{a+1}$.

Multiply and simplify:

46. $\frac{3a}{2b}$ by $\frac{2b^2}{a}$.

47. $\frac{ab}{a+b}$ by $\frac{a-b}{ab}$.

48. $\frac{ax}{1-x}$ by $\frac{1-x^2}{ax}$.

49. $\frac{a}{a-b}$ by $\frac{a-b}{a+b}$.

50. $\frac{x^2-y^2}{xy}$ by $\frac{xy^2}{x+y}$.

51. $\frac{a-x}{a^2-x^2}$ by $\frac{a^2+ax}{a-x}$.

52. $\frac{x}{a+x} \times \frac{a}{a-x} \times \frac{a^2-x^2}{x^2}$.

53. $\frac{a-x}{a+x} \times \frac{a+x}{a^2-x^2} \times \frac{a+x}{a-x}$.

Divide and simplify:

54. $\frac{3}{xy}$ by $\frac{4}{x^2y^2}$.

55. $\frac{a+b}{ab}$ by $\frac{a}{a-b}$.

56. $\frac{3a^2x}{7y}$ by $\frac{6ax^2}{21y^2}$.

57. $\frac{a}{a+b}$ by $\frac{a-b}{a^2+ab}$.

58. $\frac{a^2-b^2}{a+b}$ by $\frac{a-b}{a+b}$.

59. $\frac{x^2+1}{x^2+2x+1}$ by $\frac{x^2+1}{x+1}$.

60. $\frac{x^2-1}{x^2+2x+1} \div \frac{x-1}{x+1}$.

61. $\frac{a^2+ab}{a^2+2b+b^2} \div \frac{a^3b^2}{a^2-b^2}$.

EQUATIONS CONTAINING TWO UNKNOWN NUMBERS

40. If an equation contains two unknown numbers, these numbers may have an indefinite number of values. Thus, in the equation $x + y = 12$, the value of x may be 9, and that of y, 3; or x may be 7, and y, 5; or x may be 6, and y, 6, etc.

If two unknown numbers have each the same value in two equations, the values of the two unknown numbers may be found by so combining the two equations as to produce a third equation with only one unknown number.

Thus, let $\begin{cases} x + y = 12, & (1) \\ x - y = 2. & (2) \end{cases}$

Adding equations (1) and (2), $2x = 14$;

whence $x = 7$.

Substituting 7 for x in (1), $7 + y = 12$;

whence, $y = 5$.

Hence $x = 7$, and $y = 5$.

41. When two unknown numbers have each the same value in two given equations, the equations are said to be **simultaneous.**

The process of so combining two simultaneous equations as to produce a third equation with only one unknown number is called **elimination.**

1. Find the value of x and y in the two equations:

$$\begin{cases} 2x+2y=16, & (1) \\ 3x+y=18. & (2) \end{cases}$$

Multiplying (2) by 2, $6x+2y=36.$ (3)

Subtracting (1) from (3), $4x=20;$

whence $x=5.$

Substituting 5 for x in (2), $15+y=18;$

whence $y=3.$

Hence $x=5$, and $y=3.$

42. It is seen from the foregoing processes that one of the two unknown numbers in two simultaneous equations may be removed or eliminated *by making, if necessary, the coefficients of one of the unknown numbers the same in both equations, and then adding the resulting equations if the equal terms have unlike signs and subtracting one equation from the other if the equal terms have like signs.*

NOTE. — It is not best to teach beginners more than one method of elimination.

Find the value of x and y in the following equations:

2. $\begin{cases} 2x+y=13, \\ 3x-y=7. \end{cases}$

3. $\begin{cases} x+2y=7, \\ 2x-y=4. \end{cases}$

4. $\begin{cases} 3x-2y=11, \\ 4x+2y=24. \end{cases}$

5. $\begin{cases} 5x+2y=30, \\ 2x+2y=18. \end{cases}$

6. $\begin{cases} x+2y=18, \\ 2x-y=21. \end{cases}$

7. $\begin{cases} x+3y=36, \\ x+y=20. \end{cases}$

8. $\begin{cases} x+2y=20, \\ 2x-3y=5. \end{cases}$

9. $\begin{cases} 3x-y=11, \\ 2x+y=14. \end{cases}$

10. $\begin{cases} 4x - 3y = 12, \\ x + 3y = 18. \end{cases}$

11. $\begin{cases} x + 4y = 19, \\ 3x - 2y = 15. \end{cases}$

12. $\begin{cases} 2x + 3y = 17, \\ 3x + y = 15. \end{cases}$

13. $\begin{cases} x + 2y = 18, \\ 2x - 2y = 18. \end{cases}$

14. $\begin{cases} 5x + 3y = 26, \\ 3x - y = 10. \end{cases}$

15. $\begin{cases} 3x + 2y = 21, \\ 2x + 3y = 19. \end{cases}$

PROBLEMS

1. The sum of two numbers is 45, and their difference is 5. What are the numbers?

Let x = the larger number,

and y = the smaller number.

Then $\begin{cases} x + y = 45, \\ x - y = 5. \end{cases}$

2. The sum of two numbers is 20, and their difference is 4. What are the numbers?

3. A man is 10 years older than his wife, and the sum of their ages is 70 years. What is the age of each?

4. A and B together have 200 acres of land, and A has 40 acres more than B. How many acres has each?

5. The sum of two numbers is 75, and the first number is 15 more than twice the second. What are the numbers?

6. The sum of twice A's age and 3 times B's age is 120 years, and twice A's age is 40 years more than B's age. What is the age of each?

7. In a certain election 255 votes were cast for two candidates, and the one elected had a majority of 25. How many votes did each receive?

8. The cost of a watch and chain was \$35, and the cost of the watch was \$5 more than twice the cost of the chain. What was the cost of each?

9. A horse and carriage cost \$175, and the carriage cost \$25 more than the horse. What was the cost of each?

10. Seven years ago A's age was 3 times B's age, and 7 years hence A's age will be twice B's age. How old is each?

11. If John were 3 years older, his age would be 3 times his sister's age, but if the sister were 2 years older, her age would be $\frac{1}{2}$ of John's age. How old is each?

12. The number of pupils enrolled in a school is 82, and $\frac{2}{3}$ of the number of girls enrolled is 8 more than $\frac{1}{2}$ of the number of boys. How many of each sex are there in the school?

13. Find two numbers such that $\frac{2}{3}$ of the greater plus $\frac{3}{4}$ of the less is 16, and $\frac{2}{3}$ of the greater minus $\frac{3}{4}$ of the less is 4.

14. A flagstaff 46 feet high consists of two parts. $\frac{3}{5}$ of the lower part is 1 foot more than $\frac{2}{3}$ of the higher. What is the length of each part?

15. Find two numbers such that 5 times the first is 1 more than 3 times the second, and twice the first plus the second equals 7.

43. The value of x in a proportion may be found by writing the product of the extremes equal to the product of the means, and then finding the value of x.

1. Find the value of x in the proportion $4 : 12 :: 3 : x$.

$$\frac{4}{12} = \frac{3}{x};$$

clearing of fractions, $4x = 36$;

whence $x = 9$.

Find the value of x in the following proportions:

2. $5 : 9 :: 10 : x$.
3. $4 : 6 :: 12 : x$.
4. $x : 8 :: 6 : 12$.
5. $x : 3 :: 10 : 6$.
6. $5 : x :: 10 : 7$.
7. $4 : 6 :: x : 9$.
8. $5 : 15 :: 3 : x$.
9. $6 : 4 :: 12 : x$.
10. $x - 6 : 5 :: x + 6 : 9$.
11. $4 : 12 :: x - 4 : x + 4$.
12. $x + 3 : x - 3 :: 5 : 3$.
13. $x + 1 : x + 5 :: 4 : 6$.

PROBLEMS

1. If 5 men earn \$15 in a day, how many men will earn \$75 in a day?

Let x = number of men that will earn \$75;

then $15 : 75 :: 5 : x$.

Multiplying the extremes and the means,

$$15x = 375;$$

whence $x = 25$, number of men.

2. If 6 men can do a piece of work in 48 hours, in how many hours can 8 men do the work?

3. If 5 horses eat 10 bushels of oats in 9 days, how many bushels will they eat in 36 days?

4. If a train runs 90 miles in 3 hours, in how many hours will it run 225 miles?

5. If a man can do a piece of work in 12 days, working $8\frac{1}{3}$ hours a day, how long will it take him if he works 10 hours a day?

6. A garrison of 90 men consume 15 barrels of flour in 12 weeks. How many barrels of flour will supply them 52 weeks?

7. If a flagstaff 60 feet high casts a shadow 72 feet long, how high is a steeple whose shadow at the same time is 162 feet long?

8. If a vertical staff 4 feet long casts a shadow 5 feet long at 10 o'clock, how long at the same hour is the shadow of a flag pole 90 feet high?

9. If a pole 15 feet high casts a shadow 8 feet long at noon, how high is a tree whose shadow at the same hour is 72 feet long?

10. If a five-cent loaf of bread weighs 8 ounces when flour is \$4 a barrel, how much should it weigh when flour is \$5 a barrel?

11. A garrison of 20 men is supplied with provisions for 12 days. If 12 men leave, how long will the provisions supply the men that remain?

12. A garrison of 500 men having provisions for 60 days was reënforced after 20 days, and from that time the provisions lasted only 20 days. How many men were added to the garrison?

13. Divide 120 into two parts such that their ratio shall be as 3 to 5.

	Or,
Let x = one part;	Let $3x$ = one part;
then $120 - x$ = other part.	and $5x$ = other part.
Hence $(3 + 5) : 3 :: 120 : x$.	Then $3x + 5x = 120$;
$8x = 360$;	$x = 15$;
$x = 45$, one part;	$3x = 45$, one part;
$120 - x = 75$, other part.	$5x = 75$, other part.

14. Divide \$90 between A and B in the ratio of 4 to 5.

15. The sum of two numbers is 154, and the less is to the greater as 5 to 6. What are the numbers?

16. The difference of two numbers is 30, and the greater is to the less as 9 to 6. What are the numbers?

17. The difference between the ages of two brothers is 12 years, and the age of the elder is to the age of the younger as 7 to 5. What is the age of each?

18. A father's age is to his son's age as 7 to 3, and the sum of their ages is 60 years. What is the age of each?

19. The monthly wages of two artisans are in the ratio of 4 to 5, and one receives \$12 a month more than the other. Find their wages.

20. The length of a rectangular room is to its width as 5 to $3\frac{1}{2}$, and the length is 12 feet more than the width. Find the dimensions of the room. How many feet are in its perimeter?

21. If 24 barrels of flour will supply a garrison of 160 men, how many barrels will supply 360 men the same time?

22. If a clock ticks 120 times in a minute, how many times will it tick in 5 hours?

23. At the rate of 5 oranges for 12 cents, how many oranges can be bought for $3?

24. If 12 men can mow 20 acres of grass in a day, how many acres can 30 men mow in a day?

25. If 9 men can build a wall in 10 days, in how many days can 5 men build it?

26. If 12 men can dig a ditch in 10 days, how long will it take 5 men to dig it?

27. If 15 men can harvest a field of wheat in 6 days, how many men can harvest it in 9 days?

28. Two men built a wall for $54, and their wages were in the ratio of 4 to 5. How much did each receive?

SQUARES AND SQUARE ROOTS — AREAS

44. $3 \times 3 = 3^2$; $x \times x = x^2$; $2x \times x = 2x^2$; $x \times \frac{x}{2} = \frac{x^2}{2}$; $\frac{x}{2} \times \frac{x}{3} = \frac{x^2}{6}$.

1. What is the product of x and $3x$? $2x$ and $2x$? $2x$ and $\frac{x}{2}$? $\frac{2x}{3}$ and $3x$? $\frac{x}{2}$ and $\frac{x}{4}$? $\frac{2x}{3}$ and $\frac{3x}{4}$?

If $x = 3$, then $x \times x = 3 \times 3$; that is, $x^2 = 9$. The two members of an equation may be raised to the second power, or squared, without affecting their equality.

45. If $x^2 = 16$, then $\sqrt{x^2} = \sqrt{16}$; that is, $x = 4$. The square root of both members of an equation may be taken without affecting their equality.

Find the value of x in the following equations:

2. $x^2 = 64$.
3. $x^2 = 121$.
4. $2x^2 = 162$.
5. $9x^2 = 324$.
6. $\frac{x^2}{2} = 32$.
7. $\frac{x^2}{3} = 27$.
8. $\frac{2x^2}{3} = 96$.
9. $\frac{3x^2}{4} = 12$.
10. $3x^2 = 768$.
11. $5x^2 = 980$.
12. $6x^2 = 1014$.
13. $\frac{3x^2}{4} = 75$.
14. $\frac{3x^2}{5} = 15$.

PROBLEMS

1. One number is $\frac{2}{3}$ of another number, and their product is 54. What are the numbers?

Let $x =$ larger number;

then $\frac{2x}{3} =$ smaller number,

and $x \times \frac{2x}{3} = \frac{2x^2}{3} =$ their product.

Hence $\frac{2x^2}{3} = 54.$

Multiplying by 3, $2x^2 = 162;$

dividing by 2, $x^2 = 81;$

extracting square root, $x = 9$, larger number;

$\frac{2x}{3} = 6$, smaller number.

2. A's age is $\frac{1}{2}$ of B's age, and the product of their ages is 450 years. What are their ages?

3. The product of $\frac{2}{3}$ and $\frac{3}{4}$ of a number is 162. What is the number?

4. Two numbers are to each other as 2 to 3, and their product is 150. What are the numbers?

5. Two numbers are to each other as 3 to 5, and the difference of their squares is 256. What are the numbers?

6. A husband's age is to his wife's age as 6 to 5; $\frac{1}{3}$ of the product of their ages is 360 years. What are their ages?

7. A's age is to B's as 5 is to 2, and the difference of the squares of their ages is 525. What are their ages?

8. The width of a rectangular field is $\frac{2}{3}$ of its length, and its area is 600 square rods. What are the dimensions of the field?

9. The length of a rectangular field is $2\frac{1}{2}$ times its width, and the field contains 9 acres. What are its length and width?

10. A rectangular garden contains an acre, and its width is $\frac{2}{5}$ of its length. What are its dimensions?

11. What are the dimensions of a square garden that contains 1024 square yards?

46. The area of a triangle equals the product of its base by one half its altitude.

12. The base of a right-angled triangle is twice its altitude, and its area is 64 square inches. What are its dimensions?

SUGGESTION. — Let x = the base; then $\frac{x}{2}$ = the altitude; and $x \times \frac{x}{4} = 64$, area.

13. A triangular field contains 5 acres, and its altitude is $\frac{4}{9}$ of its base. What is its base?

14. The length of a rectangular field containing 16 acres is 10 times its width. What are its dimensions?

15. The length of a rectangular garden is to its width as 5 is to 3, and its area is 240 square rods. What are its dimensions?

16. Two numbers are to each other as 4 to 3 and their product is 192. What are the numbers?

17. The ages of a husband and wife are to each other as 5 to 4, and the product of their ages is 1280. What are their ages?

47. The area of a circle is equal to the square of its diameter multiplied by .7854.

The surface of a sphere is equal to the square of its diameter multiplied by 3.1416.

18. A circular field contains 314.16 square rods. What is its diameter?

Let x = diameter;

then $x^2 \times .7854 = 314.16.$

Dividing by .7854, $x^2 = 314.16 \div .7854 = 400;$

extracting square root, $x = 20$, number of rods in diameter.

19. A circular pond contains 1256.64 square yards. What is its diameter?

20. The surface of a globe is 7854 square inches. What is its diameter?

21. The area of a circle is 19.635 square inches. What is its diameter?

22. The surface of a globe is 706.86 square inches. What is its diameter?

23. The area of a circle is $490.8\frac{3}{4}$ square inches. What is the length of its diameter?

MISCELLANEOUS PROBLEMS

1. If the letter a represents a certain integer, what will represent the next lower integer? The next higher integer?

2. If a man earns a dollars a month, how much will he earn in b months?

3. If $\frac{5}{7}$ is added to a certain fraction, the sum will be $\frac{9}{10}$. What is the fraction?

4. The difference between $\frac{5}{6}$ and $\frac{3}{5}$ of a certain number is 14. What is the number?

5. When Charles is $\frac{2}{5}$ older than he now is, he will be 21 years of age. How old is Charles?

6. A man sold a horse for \$90, which was $\frac{1}{5}$ more than it cost him. What was the cost of the horse?

7. A lady paid \$30 for a cloak which was $\frac{3}{7}$ more than she paid for a dress. What was the cost of the dress?

8. A man paid \$150 for a watch and chain, and the chain cost $\frac{3}{7}$ as much as the watch. How much did each cost?

9. If to A's age there are added $\frac{2}{3}$ and $\frac{3}{5}$ of his age, the sum will be 68 years. What is A's age?

10. A farmer's sheep are in four fields. The first contains $\frac{2}{9}$ of all, the second $\frac{1}{6}$, the third $\frac{1}{4}$, and the fourth 52 sheep. How many sheep are in the four fields?

11. If to $\frac{5}{6}$ of a man's age 15 years are added, the sum will be $\frac{5}{4}$ of his age. How old is he?

12. The value of a house is $\frac{4}{5}$ of the value of a lot, and the value of both house and lot is $4500. What is the value of each?

13. The difference between $\frac{3}{4}$ and $\frac{3}{5}$ of B's age is 12 years. What is B's age?

14. The sum of the ages of a husband and wife is 72 years, and the wife's age is $\frac{5}{7}$ of the husband's age. What is the age of each?

15. The difference between two numbers is 45, and the less number is $\frac{4}{5}$ of the greater. What are the two numbers?

16. A field that is 80 rods long contains 10 acres. What is its width?

17. A square field contains 10 acres. How many rods are in its perimeter?

18. The length of a rectangular field is 3 times its width, and the distance around the field is 120 rods. What are its dimensions?

19. In a certain election one candidate received $\frac{3}{5}$ of the votes cast, and had a majority of 120 votes. How many votes were cast? How many did the successful candidate receive?

20. An estate of $6300 was divided among three heirs, the first receiving twice as much as the second, and the second twice as much as the third. How much did each receive?

21. The cost of a horse was $100 less than the cost of a carriage, and the carriage cost 3 times as much as the horse. What was the cost of each?

22. The fore wheel of a carriage makes 50 rotations while the hind wheel makes 30. How many rotations will the fore wheel make while the hind wheel makes 120?

23. A man owns 360 acres of land in three farms. The first contains twice as many acres as the third, and the second twice as many acres as the first and third together. How many acres does each farm contain?

24. A number is expressed by two digits whose sum is 9, and the tens' digit is twice the units' digit. What is the number?

25. A number is expressed by three digits whose sum is 10. The hundreds' digit is twice the tens' digit, and the tens' digit is 3 times the units' digit. What is the number?

26. A, B, and C have together $450; A's money is $\frac{2}{3}$ of B's, and B's is $\frac{3}{4}$ of C's. How much has each?

27. There are 360 peach, pear, and plum trees in an orchard. There are twice as many peach trees as pear trees, and 3 times as many pear trees as plum trees. How many trees of each kind are in the orchard?

28. A farm of 160 acres consists of pasture, meadow, and woodland. There is twice as much pasture as meadow, and 3 times as much meadow as woodland. How many acres of woodland are in the farm?

29. A, B, and C own together 640 acres of land. A owns twice as much as B, and B 3 times as much as C. How many acres does each own?

30. A and B start in business. A's capital is 3 times B's, and the difference in their capital is $2500. How much capital does each furnish?

31. A newsboy has $6 in dimes, quarter dollars, and half dollars. He has 5 times as many dimes as quarters, and twice as many quarters as half dollars. How many pieces of each kind has he?

32. A's farm contains $\frac{3}{4}$ as many acres as B's farm, and B's contains 40 acres more than A's. How many acres are there in each farm?

33. A, B, and C rent a pasture for $84. B pays half as much as A, and C half as much as B. How much does each pay?

34. A and B begin trade with equal capital. A draws out $33\frac{1}{3}\%$ of his capital, and B adds 50 % to his, and then B has $2000 more capital than A. What was the capital of each at first?

35. A father is 3 times as old as his son, but in 15 years he will be twice as old as his son. What is the age of each?

36. Ten years ago a mother was 4 times as old as her daughter, and now she is $2\frac{1}{2}$ times as old as her daughter. What is the age of each?

37. The time past noon is $\frac{3}{5}$ of the time to midnight. What is the time of day?

SUGGESTION. — Let x = time to midnight; then $x + \frac{3x}{5} = 12$.

38. What is the time of day when $\frac{3}{8}$ of the time past noon equals $\frac{3}{4}$ of the time to midnight?

39. The number of hours past noon is 3 times the number of hours to midnight. What is the time of day?

40. A and B can do a piece of work in 12 days, and B can do it in 20 days. In how many days can A do the work?

41. A can mow a field of grass in 6 days, and B in 9 days. They work together 3 days, when B leaves. How long will it take A to finish the mowing of the field?

42. A and B can dig a ditch in 9 days, A and C in 12 days, and B and C in 18 days. In how many days can all together dig the ditch?

43. A can do a piece of work in 6 days, and B can do $\frac{2}{3}$ as much work as A. In how many days can A and B together do the work?

44. Divide 153 into two parts such that their ratio shall be as 6 to $2\frac{1}{2}$.

45. Cut a yard of tape into two pieces such that one piece shall be 9 inches longer than the other.

46. Find two consecutive numbers whose sum is 85.

47. Find three consecutive numbers such that the first divided by 7 equals the third divided by 8.

48. A boy bought peaches at 2 for a cent, and twice as many at 3 for a cent, and then sold all at a cent apiece, and gained 44 cents. How many peaches did he sell?

49. A man sold a horse for \$90 and gained 50 %. What per cent would he have gained if he had sold the horse for \$75?

SUGGESTION. — Let x = cost of horse; then $x + \frac{x}{2} = 90$.

50. How much wheat must be taken to a mill to bring away the flour of 9 bushels, after the miller has taken out 10 % for toll?

51. If a man can do a piece of work in 15 days, working 8 hours a day, in how many days could he do the work by working 10 hours a day?

52. A pipe will fill a cistern in 4 hours and another pipe will empty it in 6 hours. If both pipes are open, how long will it take to fill the cistern?

53. The width of a room is $\frac{3}{4}$ of its length; if the width were 4 feet more and the length 4 feet less, the room would be square. What are the dimensions of the room?

54. A fruit dealer sold 5 oranges to every 3 lemons, and he sold in all 72 oranges and lemons. How many oranges and how many lemons did he sell?

55. What is the price of eggs per dozen when, with a rise of 25%, 20 dozen eggs can be bought for $5?

56. A man having $72 spent a part of it, and then had 3 times as much as he had spent. How much did he spend?

57. A's age is now twice B's age, but in 5 years A's age will be to B's as 7 to 4. What is the age of each?

58. If 12 men can build a house in 30 days, how many men more must be employed to build it in 20 days?

59. A and B had equal sums of money. A spent $15 and B $25, and then A had twice as much money as B. How much money had each at first?

60. A sold B a horse for $\frac{1}{5}$ more than its cost, and B sold it for $80, gaining $\frac{1}{3}$ of what he paid for it. How much did A pay for the horse?

61. At 2 P.M. a train running 30 miles an hour leaves Cleveland for Chicago, and at 5 P.M. an express train running 40 miles an hour leaves Cleveland for Chicago. In how many hours will the second train overtake the first?

62. A and B together own 396 acres of land, and $\frac{3}{5}$ of A's farm equals $\frac{3}{4}$ of B's. How many acres does each own?

63. A stock of goods was owned by three partners, A owning $\frac{3}{8}$, B $\frac{5}{11}$, and C the remainder; the goods were sold at a profit of $6160. What was each partner's share?

64. Five eighths of a stock of goods was destroyed by fire, and $\frac{3}{5}$ of the remainder was damaged by water.

The uninjured goods were sold at cost for $5280. What part of the goods was sold? What was the cost of the entire stock?

65. A pedestrian walked $\frac{5}{12}$ of his journey the first day, $\frac{3}{8}$ of it the second day, and then had 24 miles to travel. How long was the journey?

66. A graded school enrolls 208 boys, and $\frac{7}{15}$ of the pupils are girls. How many pupils are in the school?

67. A jeweler sold a watch for $84, gaining thereby $16\frac{2}{3}\%$. What was the cost of the watch?

68. A dealer sold a piano for $160, which was $33\frac{1}{3}\%$ less than the cost. What was its cost?

69. A man by selling a lot for $700 made $16\frac{2}{3}\%$. What was the cost of the lot? How much did he make?

70. A piece of flannel shrank $12\frac{1}{2}\%$ in fulling and then contained 42 yards. How many yards were in the piece before fulling?

71. A conductor receives $60 per month, which is 20% more than he received last year. What were his wages last year?

72. A owns 132 acres of land, which is $37\frac{1}{2}\%$ more than B owns. How many acres does B own?

73. A workman's wages were increased 20%, and he was then paid $42 per month. How much was he paid before the increase?

74. A man sold a watch for $120, which was $\frac{4}{5}$ of what it cost him. How much did it cost?

75. A farmer sold a horse for \$90, which was $\frac{1}{4}$ more than its cost. What was the cost of the horse?

76. A piece of flannel lost $\frac{2}{5}$ of its length by shrinkage in fulling, and then measured 30 yards. What was its length before fulling?

77. Three fifths of my money is in my purse, $\frac{3}{8}$ in my hand, and the remainder, which is 25 cents, is in my pocket. How much money have I?

78. A farmer sold $\frac{2}{3}$ of his sheep and then bought $\frac{3}{5}$ as many as he had left, when he had 40 sheep. How many sheep had he at first?

79. John lost $\frac{2}{5}$ of his money and spent $\frac{1}{3}$ of the remainder, and then had only 10 cents. How much money had he at first?

80. A man sold a horse for \$60, which was $\frac{4}{5}$ of $\frac{3}{4}$ of its cost. How much was lost by the bargain?

81. A man sold a horse for \$130, which was $\frac{5}{8}$ more than it cost him. What was the cost of the horse?

82. If a staff 5 feet long casts a shadow 2 feet long at 12 o'clock, what is the height of a steeple whose shadow at the same hour is 80 feet?

83. If a rod 5 feet long casts a shadow $8\frac{1}{3}$ feet long, what is the length of a pole whose shadow at the same time of day is $17\frac{1}{2}$ feet?

84. If a staff 3 feet long casts a shadow 2 feet in length, how long a shadow will a tree 90 feet high cast at the same time of day?

85. If a steeple 200 feet high casts a shadow 150 feet long, what is the height of a pole which at the same time of day casts a shadow 80 feet long?

86. If 5 men can do a piece of work in 12 days, how long will it take 6 men to do it?

87. If 8 men can do a piece of work in 15 days, how many men can do the same work in 10 days?

88. If 9 men can do a piece of work in $4\frac{2}{3}$ days, how long will it take 7 men to do it?

89. If 3 pipes will empty a cistern in 30 minutes, how many pipes will empty it in 10 minutes?

90. If it requires 12 days of 10 hours each to do a piece of work, how many days of 8 hours each will be required to do the same work?

91. If a five-cent loaf weighs 10 ounces when flour is \$4 a barrel, what ought it to weigh when flour is \$5 a barrel?

92. If 3 men can do a piece of work in $10\frac{3}{4}$ days, how long will it take 8 men to do it? 12 men?

93. If a barrel of flour will supply 12 persons $4\frac{3}{7}$ weeks, how long will it supply 7 persons? 10 persons?

94. If $12\frac{1}{2}$ tons of hay will feed 5 horses a year, how many tons will feed 8 horses a year? 12 horses?

95. A garrison of 20 men is supplied with provisions for 12 days. If 12 men leave, how long will the provisions serve the remainder?

96. If a horse eats 2 bushels of oats in 9 days, in how many days will 2 horses eat 18 bushels?

97. If 3 men can mow 18 acres of grass in 4 days, how many men can mow 9 acres in 3 days?

98. If a quantity of provisions will supply 15 men 20 days, how long will it supply 50 men?

99. If 5 men can do $\frac{3}{8}$ of a piece of work in a day, how long will it take one man to do the entire work?

100. If 8 men can do $\frac{3}{4}$ of a piece of work in 3 days, how long will it take 4 men to do the entire work?

101. If 20 men earn $120 in 4 days, how much will 5 men earn in 8 days?

102. If 5 horses eat 40 bushels of oats in 3 weeks, how many bushels will supply 12 horses?

103. If 8 men can dig a ditch 40 rods long in 6 days, how long will it take 12 men to dig a ditch 60 rods long?

104. If the interest of $50 for 9 months is $6, what will be the interest of $150 for 1 yr. 6 mo.?

105. A school enrolls 180 pupils, and the number of boys is $\frac{4}{5}$ of the number of girls. How many pupils of each sex are enrolled in the school?

106. A lady paid $130 for a watch and chain, and the cost of the chain was $\frac{5}{8}$ of the cost of the watch. What was the cost of each?

107. A man bought a watch and chain for $80, and the chain cost $\frac{1}{3}$ as much as the watch. How much did each cost?

108. A has $1\frac{1}{2}$ times as many cents as B, and they together have 40 cents. How many has each?

109. A pole 120 feet high fell and broke into two parts, and $\frac{3}{5}$ of the longer part was equal to the shorter. How long was each part?

110. A tree 120 feet in height was broken into two parts by falling, and $\frac{2}{5}$ of the shorter part equaled $\frac{2}{7}$ of the longer. What was the length of each part?

111. A person giving the time of day, said that $\frac{2}{3}$ of the time past noon equaled the time to midnight. What was the time of day?

112. What is the time of day when $\frac{3}{4}$ of the time past noon equals $\frac{3}{8}$ of the time to midnight?

113. The number of hours past noon divided by the number of hours to midnight equals 3. What is the time of day?

114. A man being asked his age, said, "Ten years ago my age was $\frac{7}{9}$ of my present age." What was his age?

115. A son's age is $\frac{3}{5}$ of the age of his father, and the sum of their ages is 80 years. What is the age of each?

116. If to my age you add its half, its third, and 28 years, the sum will be 3 times my age. What is my age?

117. Three fourths of A's age equals $\frac{4}{7}$ of B's, and the difference between their ages is 10 years. How old is each?

118. A horse cost \$90, and $\frac{3}{10}$ of the price of the horse was $\frac{3}{5}$ of 3 times the cost of the saddle. What was the cost of the saddle?

119. A man bought a horse and carriage for $280, and $\frac{2}{5}$ of the cost of the carriage was equal to $\frac{2}{3}$ of the cost of the horse. What was the cost of each?

120. A man bought a horse, saddle, and bridle for $150; the cost of the bridle was $\frac{1}{2}$ of the cost of the saddle, and the cost of the saddle was $\frac{1}{6}$ of the cost of the horse. What was the cost of each?

121. A man and his two sons earned $140 in a month; the man earned twice as much as the elder son, and the elder son earned twice as much as the younger. How much did each earn?

122. Two men hired a pasture for $40, and one put in 3 cows and the other 5 cows. How much ought each to pay?

123. A and B rent a pasture for $72; A puts in 40 sheep and B 8 cows. If 4 sheep eat as much as one cow, how much ought each to pay?

124. Two men divided a lot of wood, which they purchased together for $27; one took $5\frac{1}{2}$ cords, the other 8 cords. How much ought each to pay?

125. A and B together own 824 sheep, and A has $1\frac{2}{3}$ times as many as B. How many has each?

126. A, B, and C rent a pasture for $42; B pays half as much as A, and C half as much as B. How much does each pay?

127. A and B own a farm; A owns $\frac{3}{4}$ as much as B, and B owns 40 acres more than A. How many acres does each own?

128. $\frac{3}{4}$ of A's money is $\frac{2}{3}$ of B's, and $\frac{3}{4}$ of B's is $\frac{2}{3}$ of C's, which is \$81. How much have A and B each?

129. If a man can reap $\frac{3}{4}$ of an acre of wheat in a day, how much can 6 men reap in 10 days?

130. A makes a shoe in $\frac{2}{3}$ of a day; B makes one in $\frac{2}{5}$ of a day. How many shoes can both make in 2 days?

131. A can mow an acre of grass in $\frac{3}{4}$ of a day, and B in $\frac{3}{5}$ of a day. How long will it take both together to mow an acre?

132. Two men, A and B, agreed to build a wall for \$300; A sent 5 men for 4 days, and B 5 men for 6 days. How much ought each to receive?

133. A man can do $\frac{1}{5}$ of a piece of work in a day, and a boy can do $\frac{1}{8}$ of it in a day. In how many days can both of them, working together, do it?

134. A and B together can build a wall in 8 days, and A can build it alone in 12 days. How long will it take B to build it?

135. A can do a piece of work in 6 days, and B in 8 days. If they work together, how long will it take them to do the work?

136. John can saw a pile of wood in 6 days, and, with the assistance of Charles, he can saw it in 4 days. How long will it take Charles to saw it alone?

137. A and B can do $\frac{1}{3}$ of a piece of work in a day, and A can do $\frac{1}{5}$ of it in a day. How long will it take B alone to do it?

138. A can do a piece of work in 4 days, B in 5 days, and C in 6 days. In what time can they together do it?

139. A and B can do a piece of work in 10 days, and A, B, and C in 8 days. How long will it take C alone to do the work?

140. A, B, and C can do a job in 20 days; A and B can do it in 40 days; and A and C in 30 days. In how many days can each do it alone?

141. If 12 men can dig a ditch in 20 days, how long will it take to dig the ditch if 3 more men are employed?

142. Two men start from two places 495 miles apart, and travel toward each other; one travels 20 miles a day, and the other 25 miles a day. In how many days will they meet?

143. A owes $\frac{2}{3}$ of B's income, but, by saving $\frac{7}{45}$ of B's income annually, he can pay his debt in 5 years, and have \$50 left. What is B's income?

144. If a man traveling 14 hours a day makes half a journey in 5 days, how long will it take to make the other half, if he travels 10 hours a day?

145. If a man can do a piece of work in $9\frac{4}{5}$ days, working 8 hours a day, how long will it take, if he works 6 hours a day?

146. The sum of two numbers is 60, and if the greater number be divided by the less, the quotient will be 3. What are the numbers?

147. An orchard contains 1225 trees in equal rows, and it has as many rows of trees as there are trees in

each row. How many rows of trees are in the orchard? How many trees are in each row?

148. If a steamer sails 9 miles an hour down stream, and 5 miles an hour up stream, how far can it go down stream and back again in 14 hours?

149. A steamer sails a mile down stream in 5 minutes, and a mile up stream in 7 minutes. How far down stream can it go and return again in one hour?

150. A and B did a piece of work, and $\frac{2}{5}$ of what A did equaled $\frac{1}{5}$ of what B did. If B received \$18, how much did A receive?

151. A man after spending $\frac{3}{4}$ of his money earned $\frac{1}{2}$ as much as he had spent, and then had \$21 less than he had at first. How much money had he at first?

152. A pipe will fill a cistern in 4 hours, and another will fill it in 6 hours. How long will it take to fill it when both pipes are open?

153. A pipe will fill a tank with water in 3 hours, and a second pipe will fill it in 4 hours. In what time will both together fill it?

154. An estate was so divided between two heirs that $\frac{2}{3}$ of the share of the elder was equal to $\frac{3}{4}$ of the share of the younger, and the difference between their shares was \$200. What was the share of each?

155. A and B are partners in business; A's capital is equal to $\frac{3}{4}$ of B's, and their profits, which are \$2100, are divided on the basis of the capital invested. What is the share of each?

156. A and B are partners; $\frac{2}{5}$ of A's capital is equal to $\frac{3}{8}$ of B's, and their loss in business is $2600. What is each partner's share of the loss?

157. A, B, and C are partners; B has invested $\frac{3}{4}$ as much capital as A, and C $\frac{2}{3}$ as much as B. If their profits amount to $6300, what will be each partner's share?

158. Seven years ago A's age was 7 times B's age, and 3 years hence A's age will be 3 times B's. What is the age of each?

159. Two railroad trains start from two cities 495 miles apart, and run toward each other on the same track, one running 30 miles an hour and the other 25 miles an hour. In how many hours will they meet?

160. A train of cars running 25 miles an hour is followed by another train running 30 miles an hour. If the first train has 3 hours the start, in how many hours will the second train overtake it?

ANSWERS

Page 14. — **3.** Vest $4, coat $20. **4.** 9, 36. **5.** 9, 27. **6.** 9 yd., 45 yd. **7.** A 100 sheep, B 50. **8.** F. 42, e. s. 14, y. s. 7. **9.** 21. **10.** A $120, B $240, C $480. **11.** A $4, B $2, C $12. **12.** A $4000, B $2000, C $10000. **13.** Boys 80, girls 160.

Page 15. — **14.** $60 buggy, $120 horse. **15.** 186. **16.** C 18, B 36, A 72. **17.** 7 yd., 14 yd., 21 yd. **18.** Sons $3000, w. $12000. **19.** B $2000, A $4000, C $6000. **20.** 1st yr. $900, 2d $1800, 3d $3600. **21.** F. $80, e. s. $40, y. s. $20.

Page 16. — **22.** Corn 120 bu., wheat 240 bu., oats 360 bu. **24.** Q. 10, d. 20, n. 60. **25.** 36 pieces. **26.** Q. 4, d. 16, n. 32.

Page 17. — **28.** F. 45, s. 15. **29.** 135. **30.** Saddle $6, harness $18. **31.** M. 45, d. 15. **32.** Nephew 5, uncle 45. **33.** 15. **34.** 14.

Page 18. — **35.** Boys 80, girls 100. **36.** 22 ft., 38 ft. **37.** 45 ft., 75 ft. **38.** A 40, B 20, C 35. **39.** $60, $80. **40.** 50, 70.

Page 19. — **41.** Ch. $20, w. $65. **42.** 158, 206. **43.** M. 50, w. 65, ch. 230. **44.** $400, $600, $800. **45.** 16, 34. **46.** 35 ft., 55 ft. **47.** A $3500, B $2000. **48.** 24 in., 40 in. **49.** 1st c. 12, 2d 24, 3d 40.

Page 20. — **50.** F. 48, s. 24. **51.** M. 35, d. 15. **52.** 21, 51. **53.** 5. **54.** F. 40, s. 10. **55.** A 32 sheep, B 16, C 27. **56.** 79 A., 121 A. **57.** A 30, B 40, C 50. **58.** 70 A., 110 A. **59.** A $50, B $130.

Page 21. — **60.** Serge $11.25, silk $27. **61.** 4 doz. or., 8 doz. lem., 16 doz. pears. **62.** Vest $5, tr. $10, coat $15. **63.** Bon. $5, dr. $13, cl. $17. **64.** $25, $40, $35. **65.** 9 hours; 27 m., 36 m.

Page 22. — **2.** $x = 18$. **3.** $x = 24$. **4.** $x = 12$. **5.** $x = 12$. **6.** $x = 40$. **7.** $x = 1\frac{1}{5}$. **8.** $x = 10$. **9.** $x = 12$. **10.** $x = 8$. **11.** $x = 36$. **12.** $x = 12$. **13.** $x = 12$. **14.** $x = 8$. **15.** $x = 24$. **16.** $x = 12$. **17.** $x = 60$. **18.** $x = 4$. **19.** $x = 18$. **20.** $x = 6$.

Page 23. — **2.** 18, 30. **3.** 18, 12, 6. **4.** 288. **5.** 24 yr. **6.** 15 yr. **7.** 24 ct.

Page 24. — **8.** 120 sheep. **9.** 80 sheep. **10.** 60 yd. **11.** Horse \$70, cow \$20. **12.** 30 rd. by 10 rd. **13.** 75 m. **14.** H. \$100, c. \$150. **15.** 1st \$60, 2d \$100, 3d \$80. **16.** A's 30, B's 40. **17.** H's 42, w's 30.

Page 25. — **18.** A's 10, B's 25, C's 30. **19.** A 90 A., B 54 A. **20.** 40, 30. **21.** 15, 30. **22.** 20. **23.** W. \$60, ch. \$15. **24.** \$160. **25.** A \$1200, B \$3600, C \$2400. **26.** W. \$4500, s. \$3000; \$9000.

Page 26. — **27.** 1st 60 A., 2d 39 A., 3d 52 A. **28.** 240 bu. **29.** 50 ft., 80 ft. **30.** \$80. **31.** \$75. **32.** 49, 50. **33.** A \$70, B \$55. **34.** 30, 50 or 270, 250. **35.** A 200, B 80, C 60.

Page 27. — **36.** F. 40 yr., s. 16 yr. **37.** 50 yr. **38.** Apples 25 bu., peaches 10 bu., pears 15 bu. **39.** 1st 39 bu., 2d 52 bu., 3d 26 bu. **40.** 2-cent, 20; 5-cent, 10; 1-cent, 10.

Page 28. — **42.** $3\frac{3}{7}$ days. **43.** $3\frac{1}{3}$ days. **44.** $1\frac{1}{13}$ days. **45.** $2\frac{10}{13}$ days. **46.** $4\frac{4}{5}$ hr.

Page 29. — **48.** 40 days. **49.** 18 days. **50.** 15 days. **51.** 30 men. **52.** 3 days. **53.** 1st 80 gal., 2d 40 gal., 3d 20 gal. **54.** 1st 4, 2d 8, 3d 24. **55.** \$7500. **56.** 1st 60, 2d 90, 3d 100.

Page 30. — **57.** 8 P.M. **58.** 12 in. by 8 in. **59.** 36. **60.** 54. **61.** A's 20 yr., B's 10 yr., C's 40 yr. **62.** 77 in. **63.** 25, 20. **64.** 15. **65.** 7800. **66.** Elder \$10000, younger \$7000. **67.** \$3240, \$4860.

Page 31. — **68.** 20 ft. **69.** $42\frac{21}{41}$ ft. **70.** 15 yr. **71.** 42 yr. **72.** \$6000. **73.** 50 ft., 30 ft. **74.** H. \$150, c. \$200. **75.** \$10. **76.** A's \$1800, B's \$3000.

Page 34. — **15.** $a + b + c + d$. **16.** $a - b + c$. **17.** $10a - b - 2c$. **18.** $16x^2y$. **19.** $2a^2b - 10ab$. **20.** $a^2 + 5ab - b^2$. **21.** $4a^2b + 3ab^2$. **22.** $2x^2y^2 - 3xy$. **23.** $a^2bc + 3b$. **25.** $17ab - 19ac + 5bc$.

Page 35. — **26.** $6ab + 4b - 9c$. **27.** $2ax^2 - 15ax$. **28.** $6a^2b^2x^2 - abc + xy$. **29.** $10a^2m^2n - 5a^2m^2n^2 + 3xy$. **30.** $10mnx^2 + 8mx^2$. **31.** $6a^2b^2 + 4ab + 4ac$. **32.** $3a^2x^2 + abx$. **33.** $4(a + b)$. **34.** $7(x + y) - 3(x - y)$, or $4x + 10y$. **35.** $-ax^2$. **36.** $4ax + b^2 + ac$.

Page 37. — **12.** $2a - 3b - 2c$. **13.** $5a^2 - 10ab + 4c$.

Page 38. — **14.** d. **15.** $14x^2 - 7xy - 2$. **16.** $2b$. **17.** $3a^2b - bc^2$. **18.** $x^2 + 4xy - y^2$. **19.** $-3ax^2 + 2bx$. **20.** $a^2x^2 + 10ax$. **21.** $x^2y + 7ab$.

Page 39. — **22.** $8x - 4$. **23.** $2x + 3$. **24.** $2a - 3$. **25.** $3a + 3$. **26.** $5x - 1$. **27.** 10. **28.** $x + \frac{1}{4}$. **29.** $-c$. **30.** $4xy - y^2$. **31.** ab. **32.** $5a^2 - (2b^2 + 3ab - c^2)$. **33.** $x^2 - (2xy - y^2) - 5$. **34.** $ab - (a^2 - 2ab + b^2)$. **35.** $a^3 - 3a^2b - (3ab^2 + b^3)$. **36.** $x = 8$. **37.** $x = 5$. **38.** $x = 4$. **39.** $x = 10$.

Page 40. — **40.** $x = 21$. **41.** $x = 10$. **42.** $x = 7$. **43.** $x = 6$. **44.** $x = 3$. **45.** $x = 6$. **46.** $x = 7$. **47.** $x = 2$. **48.** $x = 30$. **49.** $x = 10$. **50.** $x = 20$. **51.** $x = 2$. **52.** $x = 7$. **53.** $x = 3$. **54.** $x = 1$. **56.** $x = 15$. **57.** $x = 10$. **58.** $x = 20$.

Page 42. — **8.** $12a^3b^3x^3$. **9.** $-12a^3b^3x^5$. **10.** $-36a^4bx^4y^2$. **11.** $35a^3b^3c^3$. **12.** $12x^3y^3$. **13.** $-2a^5b^3x^3$. **14.** $-6a^4x^4y^5$. **15.** $3a^3x^3y^4$.

Page 43. — **17.** $-4a^3b + 8a^2b^2 - 4ab^3$. **18.** $12x^4y^2 - 8x^3y^3 + 20x^2y^4 + 4xy^5$. **19.** $-36x^2y^3 + 12x^3y^2 - 6x^4y - 3x^5y$. **20.** $-3x^3y^2 - 15x^4y^4 + 6x^2y^5$. **21.** $-2a^3b^2 + 4a^2b^3 - 2ab^4$. **22.** $6x^4y^2 - 9x^3y^3 - 3x^2y^4$. **23.** $-5x^5y^2 + 10x^4y^2 - 25x^3y^2 + 20x^2y^2$. **24.** $-12x^4y^2 - 4x^3y^3 + 24x^2y^2$. **25.** $2xy^4 - 2x^2y^3 + x^3y^2$.

Page 44. — **27.** $a^4 - 2a^3b + 2ab^3 - b^4$. **28.** $x^6 - 4x^5 + 3x^4 + 2x^3 + 4x^2 - 15$. **29.** $3a^4 - 10a^3b + 10a^2b^2 - 4ab^3$. **30.** $x^4 - 2x^2y^2 + y^4$. **31.** $x^5 - 41x - 120$. **32.** $x^4 + 9x^2 + 81$. **33.** $x^6 - 34x^4 - 48x^2 + 4x + 20$. **34.** $x^5 - 5x^4 + 15x^3 - 35x^2 + 24x$. **35.** $1 + x - x^3 - x^4$. **36.** $x^2 - y^2 - 2y - 1$. **37.** $x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$. **38.** $a^3 + 3a^2b + 3ab^2 + b^3$. **39.** $a^2 + 2a - b^2 + 1$. **40.** $1 + 3x + 3x^2 + x^3$. **41.** $a^5 - a^2b^3 - a^3b^2 + b^5$. **42.** $4a^2b^2 - 9c^2$. **43.** $4a^2 - 4b^2$. **44.** $5a^2 - 2ax - 3x^2$. **45.** $9x^2 + 3xy - 6x - 2y$.

Page 46. — **3.** $-7x^3y^2$. **4.** $-8a^4b^3$. **5.** $-3b^2c^2$. **6.** $7a^2$. **7.** $-2ac$. **8.** $7x^2y^2$. **9.** $-3a^3x$. **10.** $-8yz$. **11.** $-3m^2n^2$. **12.** $-6a^3xy$. **13.** $7a^2b^2x^2$. **14.** $-3mnx$. **15.** $8abx$.

Page 47. — **17.** $3ax^2 - 2xy^2 + a^2y^3$. **18.** $-5x + 4y - 5z$. **19.** $-5abc + 4b^2 + 3$. **21.** $a + b$. **22.** $a - b$. **23.** $a + b$. **24.** $2a - b$. **25.** $3x + 2y$. **26.** $x^2 - y^2$. **27.** $x^2 - y^2$. **28.** $1 - x$. **29.** $1 + x + 2x^2$.

Page 48. — **30.** $a - b$. **31.** $x + 8$. **32.** $x^2 + xy + y^2$. **33.** $2a - 3b$. **34.** $1 + 2x + x^2$. **35.** $x^4 + y^4$. **36.** $a^2 - b^2$. **37.** $a^2 + ab + b^2$.

38. x^2-5x+4. **39.** $x+a$. **40.** $a-b$. **41.** $2x^2-5$. **42.** $5x^2-3x+9$. **43.** x^2+x+2. **44.** x^2-2x+3. **45.** $a-b$. **46.** $a-c$. **47.** x^2-3x-1.

Page 50.—23. $4x^2-y^2$. **24.** $4x^2-9y^2$. **25.** a^2-4b^2. **26.** $a^2x^2-9b^2$. **31.** $(3x+2y)(3x-2y)$. **32.** $\left(\frac{a}{2}+2b\right)\left(\frac{a}{2}-2b\right)$. **33.** $(ax+3b)(ax-3b)$. **34.** $(4ax^2+3)(4ax^2-3)$.

Page 51.—36. $2x(2a+3xy)(2a-3xy)$. **37.** $3x^2y^2(x+3y)(x-3y)$. **38.** $x(2x+3)(2x-3)$. **43.** $25+10x+x^2$. **44.** $9a^2+6ab+b^2$. **45.** $4x^2+4xy+y^2$. **46.** $a^2x^2+6ax+9$. **47.** $9x^2+12xy+4y^2$. **52.** $a^4-2a^2b^2+b^4$. **53.** $9a^2-6ab+b^2$. **54.** $x^2-6xy+9y^2$. **55.** $a^2x^2-8ax+16$. **56.** $4a^2-12ab+9b^2$.

Page 52.—58. $(a+3b)(a+3b)$. **59.** $(a^2-b)(a^2-b)$. **60.** $(1-5ab^2)(1-5ab^2)$. **61.** $(2y-1)(2y-1)$. **62.** $(5x+y^2)(5x+y^2)$. **63.** $(xy-5)(xy-5)$. **65.** $2x(x-2)(x-2)$. **66.** $x(x-5)(x-5)$. **67.** $5a(a-b)(a-b)$. **68.** $2(a^2+c^2)(a^2+c^2)$. **69.** $3(x^2+2)(x^2+2)$. **70.** $4(1-5ab)(1-5ab)$.

Page 53.—1. $x^2+7x+12$. **2.** $x^2+2x-15$. **3.** $x^2-3x-10$. **4.** $x^2-9x+20$. **5.** $x^2-4x-21$. **6.** $x^2+(a+b)x+ab$. **7.** $x^2-(a-b)x-ab$. **8.** $x^2-(a+b)x+ab$. **9.** $x^2+cx-6c^2$. **10.** $x^2+2bx-15b^2$.

Page 54.—13. $(x+4)(x+4)$. **14.** $(x+2)(x+5)$. **15.** $(x-6)(x+3)$. **16.** $(x+9)(x-7)$. **17.** $(x-10)(x+7)$. **18.** $(x-8)(x-7)$. **19.** $(x+7)(x-4)$. **20.** $(x+9)(x-2)$. **21.** $(x+7)(x-6)$. **22.** $(x-7)(x-4)$. **23.** $(x+4)(x+3)$. **24.** $(x-5)(x-7)$. **25.** $(x+8)(x-3)$. **26.** $(x-12)(x+5)$. **27.** $(x-9)(x+5)$. **28.** $(x+5)(x-3)$. **29.** $(x-7)(x-2)$. **30.** $(x-5)(x+1)$. **31.** $(x-6)(x-4)$. **32.** $(x-3)(x+2)$. **33.** $(x-4)(x+2)$. **34.** $(x-7)(x+4)$. **35.** $(x-6)(x-1)$. **36.** $(x-4)(x+3)$. **37.** $(x-5)(x+3)$. **38.** $(x-7)(x-3)$. **39.** $(x-8)(x-4)$. **40.** $(x-8)(x-7)$. **41.** $(x-10)(x+6)$. **42.** $(x+5)(x-2)$.

Page 55.—1. $(x+5)(x-5)$. **2.** $(4+a)(4-a)$. **3.** $(1+3a)(1+3a)$. **4.** $(x-1)(x-1)$. **5.** $3x(x+y)(x-y)$. **6.** $x(x-5)(x-5)$. **7.** $2x(x+2)(x+2)$. **8.** $9(a^2+2)(a^2-2)$. **9.** $(a^2+b^2)(a+b)(a-b)$. **10.** $(2x-1)(2x-1)$. **11.** $(a+b)(a-b)(a+b)(a-b)$. **12.** $(a+2b^2)(a-2b^2)$. **13.** $(x+4)(x+4)$. **14.** $(x-ab)$

$(x - ab)$. **15.** $5b(a-2b)(a-2b)$. **16.** $(x+2)(x-7)$. **17.** $(x-9)(x-2)$. **18.** $(x+5)(x+1)$. **19.** $(10-x)(10-x)$. **20.** $3(4-y)(4-y)$. **21.** $3(x-y)(x-y)$. **22.** $7(x+3)(x-3)$. **23.** $(x-2)(x-3)$. **24.** $(3a-b)(3a-b)$. **25.** $(x+4)(x-4)$. **26.** $(5+x)(5-x)$. **27.** $(3x+5)(3x-5)$. **28.** $(2x+3)(2x-3)$. **29.** $9(2+a)(2-a)$. **30.** $(x-y)(x-y)$. **31.** $(1-4a)(1-4a)$. **32.** $(2a+b)(2a+b)$. **33.** $(a^2+2b^2)(a^2-2b^2)$. **34.** $5(x+3)(x-3)$. **35.** $4(x+y)(x+y)$. **36.** $2(2x+5)(2x-5)$. **37.** $5(x-2y)(x-2y)$. **38.** $5(1-2y)(1-2y)$. **39.** $(x-5)(x+3)$. **40.** $(x-9)(x+2)$. **41.** $3(a-1)(a-1)$. **42.** $(x+3)(x+2)$. **43.** $(x-3)(x+1)$. **44.** $(x-15)(x+4)$. **45.** $(x+9)(x-2)$. **46.** $(x-8)(x+3)$.

Page 56. — **1.** $\frac{ab}{5}$. **2.** $\frac{1}{a^2-2}$. **3.** $\frac{a-b}{a}$. **4.** $\frac{a-b}{a+b}$. **5.** $\frac{a+b}{2a}$. **6.** $\frac{x-y}{x+y}$. **7.** $a-b$. **8.** $b+\frac{x}{a}$. **9.** $x-y$. **10.** $x+y$. **11.** $x+7$. **12.** $1-\frac{x^2}{ab}$.

Page 57. — **14.** $\frac{3x^2-8x}{x-3}$. **15.** $\frac{14}{x+7}$. **16.** $\frac{2a^2+ax+x^2}{a+x}$. **17.** $\frac{a^2+x^2}{a+x}$. **18.** $\frac{2a^2}{a+b}$. **19.** $\frac{2ax-4a}{x-1}$. **21.** $\frac{a^2}{abc}, \frac{b^2}{abc}, \frac{c^2}{abc}$. **22.** $\frac{4a^2}{12ac}, \frac{3b}{12ac}, \frac{2c^2}{12ac}$. **23.** $\frac{6a}{10a^2}, \frac{25}{10a^2}, \frac{4a}{10a^2}$. **24.** $\frac{a^2-2ab+b^2}{a^2-b^2}, \frac{a^2+2ab+b^2}{a^2-b^2}$. **25.** $\frac{x-1}{x^2-1}, \frac{1}{x^2-1}, \frac{x+1}{x^2-1}$. **26.** $\frac{a^2-b^2}{a(a+b)}, \frac{a^2-ab}{a(a+b)}, \frac{a^2+2ab+b^2}{a(a+b)}$.

Page 58. — **28.** $\frac{2x}{x^2-y^2}$. **29.** $\frac{2x^2+2y^2}{x^2-y^2}$. **30.** $\frac{2x-1}{x^2-5x+6}$. **31.** $\frac{1}{a-b}$. **32.** $\frac{a^2b}{a^2-b^2}$. **33.** $\frac{5-3x}{xy}$. **34.** $\frac{x+1}{x^2}$. **35.** $\frac{x^2-y^2}{xy}$. **36.** $\frac{5x^2-13x+8}{x^2+x-12}$. **37.** $\frac{14x+2}{15+2x-x^2}$. **38.** $\frac{-b^2}{a^2+ab}$. **39.** $\frac{2b}{a^2-b^2}$. **40.** $\frac{ab}{a^2-b^2}$. **41.** $\frac{8a}{1-4a^2}$. **42.** $\frac{2}{x-a}$. **43.** $\frac{a+b-2ab^2}{a^2-b^2}$. **44.** $\frac{2}{1-x}$. **45.** $\frac{4a+8}{a^2-1}$.

Page 59. —**46.** $3b$. **47.** $\frac{a-b}{a+b}$. **48.** $1+x$. **49.** $\frac{a}{a+b}$. **50.** $xy-y^2$. **51.** $\frac{a}{a-x}$. **52.** $\frac{a}{x}$. **53.** $\frac{1}{a-x}$. **54.** $\frac{3xy}{4}$. **55.** $\frac{a^2-b^2}{a^2b}$. **56.** $\frac{3ay}{2x}$. **57.** $\frac{a^2}{a-b}$. **58.** $a+b$. **59.** $\frac{1}{x+1}$. **60.** 1. **61.** $\frac{a-b}{a^2b^2}$.

Page 61. —**2.** $x=4$; $y=5$. **3.** $x=3$; $y=2$. **4.** $x=5$; $y=2$. **5.** $x=4$; $y=5$. **6.** $x=12$; $y=3$. **7.** $x=12$; $y=8$. **8.** $x=10$; $y=5$. **9.** $x=5$; $y=4$.

Page 62. —**10.** $x=6$; $y=4$. **11.** $x=7$; $y=3$. **12.** $x=4$; $y=3$. **13.** $x=12$; $y=3$. **14.** $x=4$; $y=2$. **15.** $x=5$; $y=3$. **2.** 12, 8. **3.** Man's 40, wife's 30. **4.** A 120 A., B 80 A. **5.** 1st 55, 2d 20. **6.** A's 30 yr., B's 20 yr.

Page 63. —**7.** 140, 115. **8.** W. \$25, ch. \$10. **9.** C. \$100, h. \$75. **10.** A's 49 yr., B's 21 yr. **11.** J. 18 yr., S. 7 yr. **12.** 42 girls, 40 boys. **13.** 15, 8. **14.** L. 25 ft., up 21 ft. **15.** 1st 2, 2d 3.

Page 64. —**2.** $x=18$. **3.** $x=18$. **4.** $x=4$. **5.** $x=5$. **6.** $x=3\frac{1}{2}$. **7.** $x=6$. **8.** $x=9$. **9.** $x=8$. **10.** $x=21$. **11.** $x=8$. **12.** $x=12$. **13.** $x=7$.

Page 65. —**2.** 36 hr. **3.** 40 bu. **4.** $7\frac{1}{2}$ hr. **5.** 10 days. **6.** 65 bbl. **7.** 135 ft. **8.** $112\frac{1}{2}$ ft. **9.** 135 ft. **10.** $6\frac{2}{3}$ oz. **11.** 30 days.

Page 66. —**12.** 500 men. **14.** A \$40, B \$50. **15.** 70, 84. **16.** 90, 60. **17.** E. 42 yr., y. 30 yr. **18.** F. 42 yr., s. 18 yr. **19.** 1st \$48, 2d \$60.

Page 67. —**20.** L. 40 ft., w. 28 ft.; pe. 136 ft. **21.** 54 bbl. **22.** 36000 t. **23.** 125 oz. **24.** 50 A. **25.** 18 days. **26.** $14\frac{2}{3}$ days. **27.** 10 men. **28.** 1st \$24, 2d \$30.

Page 68. —**2.** $x=8$. **3.** $x=11$. **4.** $x=9$. **5.** $x=6$. **6.** $x=8$. **7.** $x=9$. **8.** $x=12$. **9.** $x=4$. **10.** $x=16$. **11.** $x=14$. **12.** $x=13$. **13.** $x=10$. **14.** $x=5$.

Page 69. —**2.** A's 15 yr., B's 30 yr. **3.** 18. **4.** 10, 15. **5.** 12, 20. **6.** H's 36 yr., w's 30 yr.

Page 70. — **7.** A's 25 yr., B's 10 yr. **8.** L. 30 rd., w. 20 rd. **9.** L. 60 rd., w. 24 rd. **10.** L. 20 rd., w. 8 rd. **11.** 32 yd. by 32 yd. **12.** B. 16 in., al. 8 in. **13.** B. 60 rd., al. $26\frac{2}{3}$ rd. **14.** L. 160 rd., w. 16 rd. **15.** L. 20 rd., w. 12 rd.

Page 71. — **16.** 16, 12. **17.** H's 40 yr., w's 32 yr. **19.** 40 yd. **20.** 50 in. **21.** 5 in. **22.** 15 in. **23.** 25 in.

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Page 72. — **3.** $\frac{1}{16}$. **4.** 60. **5.** 15 yr. **6.** \$75. **7.** \$21. **8.** W. \$105; ch. \$45. **9.** 30 yr.

Page 73. — **10.** 144 sheep. **11.** 36 yr. **12.** L. \$2500, h. \$2000. **13.** 80 yr. **14.** H. 42 yr., w. 30 yr. **15.** 180, 225. **16.** 20 rd. **17.** 160 rd. **18.** L. 45 rd., w. 15 rd. **19.** 600 votes; 360 votes.

Page 74. — **20.** 1st \$3600, 2d \$1800, 3d \$900. **21.** H. \$50, c. \$150. **22.** 200 ro. **23.** 1st 80 A., 2d 240 A., 3d 40 A. **24.** 63. **25.** 631. **26.** A \$100, B \$150, C \$200. **27.** Plum 36, pear 108, peach 216.

Page 75. — **28.** 16 A. **29.** A 384 A., B 192 A., C 64 A. **30.** A's \$3750, B's \$1250. **31.** 3 half dollars, 6 quarters, 30 dimes. **32.** A's 120 A., B's 160 A. **33.** A \$48, B \$24, C \$12. **34.** \$2400. **35.** F's 45 yr., s's 15 yr.

Page 76. — **36.** M's 50 yr., d's 20 yr. **37.** 4.30 P.M. **38.** 8 P.M. **39.** 9 P.M. **40.** 30 days. **41.** 1 day. **42.** 8 days. **43.** $3\frac{3}{5}$ days. **44.** 108, 45. **45.** $13\frac{1}{2}$ in., $22\frac{1}{2}$ in.

Page 77. — **46.** 42, 43. **47.** 14, 15, 16. **48.** 72 peaches. **49.** 25%. **50.** 10 bu. **51.** 12 days. **52.** 12 hr. **53.** L. 32 ft., w. 24 ft. **54.** 45 or., 27 lem.

Page 78. — **55.** 20 ct. **56.** \$18. **57.** A's 30 yr., B's 15 yr. **58.** 6 men. **59.** \$35. **60.** \$50. **61.** 9 hr. **62.** A 220 A., B 176 A. **63.** A's \$2310, B's \$2800, C's \$1050. **64.** $\frac{3}{20}$, or 15%; \$35200.

Page 79. — **65.** $115\frac{1}{2}$ miles. **66.** 390 pupils. **67.** \$72. **68.** \$240. **69.** \$600; \$100. **70.** 48 yd. **71.** \$50. **72.** 96 A. **73.** \$35. **74.** \$150.

Page 80. — **75.** \$72. **76.** 50 yd. **77.** \$10. **78.** 75 sheep. **79.** 25 ct. **80.** \$40 lost. **81.** \$80. **82.** 200 ft. **83.** $10\frac{1}{2}$ ft. **84.** 60 ft.

Page 81. — **85.** $106\frac{2}{3}$ ft. **86.** 10 days. **87.** 12 men. **88.** 6 days. **89.** 9 pipes. **90.** 15 days. **91.** 8 oz. **92.** $2\frac{1}{16}$ days. **93.** $5\frac{11}{35}$ w. **94.** 30 tons. **95.** 30 days. **96.** $40\frac{1}{2}$ days.

Page 82. — **97.** 2 men. **98.** 6 days. **99.** $13\frac{1}{3}$ days. **100.** 8 days. **101.** \$60. **102.** 96 bu. **103.** 6 days. **104.** \$36. **105.** 80 boys, 100 girls. **106.** W. \$80, ch. \$50. **107.** W. \$60, ch. \$20. **108.** A 24 ct., B 16 ct.

Page 83. — **109.** L. 75 ft., s. 45 ft. **110.** L. 70 ft., s. 50 ft. **111.** 7 : 12 P.M. **112.** 4 P.M. **113.** 9 P.M. **114.** 45 yr. **115.** F's 50 yr., s's 30 yr. **116.** 24 yr. **117.** A's 32 yr., B's 42 yr. **118.** \$15.

Page 84. — **119.** C. \$175, h. \$105. **120.** Br. \$10, sad. \$20, hs. \$120. **121.** Y. s. \$20, e. s. \$40, f. \$80. **122.** \$15, \$25. **123.** A \$40, B \$32. **124.** \$11, \$16. **125.** A 515 sheep, B 309 sheep. **126.** A \$24, B \$12, C \$6. **127.** A 120 A., B 160 A.

Page 85. — **128.** A \$64, B \$72. **129.** 45 A. **130.** 8 shoes. **131.** $\frac{1}{3}$ day. **132.** A \$120, B \$180. **133.** $3\frac{1}{13}$ days. **134.** 24 days. **135.** $3\frac{3}{7}$ days. **136.** 12 days. **137.** $7\frac{1}{2}$ days.

Page 86. — **138.** $1\frac{23}{37}$ days. **139.** 40 days. **140.** A 120 days, B 60 days, C 40 days. **141.** 16 days. **142.** 11 days. **143.** \$450. **144.** 7 days. **145.** $13\frac{1}{15}$ days. **146.** L. 15, gr. 45. **147.** 35 trees.

Page 87. — **148.** 45 miles. **149.** 5 miles. **150.** \$36. **151.** \$56. **152.** $2\frac{2}{5}$ hr. **153.** $1\frac{5}{7}$ hr. **154.** E's \$1800, y's \$1600. **155.** A's \$1200, B's \$900.

Page 88. — **156.** A's \$1000, B's \$1600. **157.** A's \$2800, B's \$2100, C's \$1400. **158.** A's 42 yr., B's 12 yr. **159.** 9 hours. **160.** 15 hours.